

UA2 Homework 4

4.1 Prove that TFAE for every finite idempotent \underline{A} and every $n \geq 2$.

(i) $\text{Clo}(\underline{A})$ contains an n -ary cyclic operation
(i.e., $\forall a_1, \dots, a_n \in A \ t(a_1, a_2, \dots, a_n) = t(a_2, \dots, a_n, a_1)$).

(ii) Each $R \leq \underline{A}^n$ which is invariant under cyclic shift (i.e. $(a_1, \dots, a_n) \in R \Rightarrow (a_2, \dots, a_n, a_1) \in R$) contains a constant tuple (i.e. $\exists a (a, a, \dots, a) \in R$)

(Hint: for the interesting implication, define R as the subuniverse generated by cyclic shifts of a tuple and get an operation that behaves like cyclic for this tuple. Compose such operations together in a smart way.)

4.2 Let $\underline{A} := (\mathbb{N}, \text{all injective operations})$, and $\mathcal{A} = \text{Clo}(\underline{A})$

(i) Characterize members of \mathcal{A} .

(ii) Prove that \mathcal{A} contains no Taylor operation

(iii) Show that $\exists f, u \in \mathcal{A} \ f(y, x) \approx u(f(x, y))$

(iv) Find nontrivial height 1 identities satisfied by some operations of \mathcal{A}