

## UA2 Homework 2

2.1 Find a convergent rewriting system equivalent to the identity  $(x \cdot y) \cdot (y \cdot z) \approx y$ .

(Hint: Knuth-Bendix will produce 2 additional rules)

(Remark: use the concepts informally.)

E.g. do not formally prove that certain substitution is a most general unifier)

2.2 Prove that  $\mathcal{E}$  is finitely terminating.

$$\mathcal{E} = \left\{ \begin{array}{l} s(x) + (y + z) \approx x + (s(s(y)) + z), \\ s(x) + (y + (z + w)) \approx x + (z + (y + w)) \end{array} \right\}$$

Signature is  $\Sigma = \{ \text{nullary } 0, \text{unary } s, \text{binary } + \}$ .

(Hint: search the internet for useful reduction orders)

(Remark: the system is interesting in that derivations can be very long: If  $f(n)$  denotes the length of the longest path in  $D(\mathcal{E})$  from a term of length  $n$  to a terminal vertex, then  $f$  cannot be bounded from above by any primitive recursive function.)