# UA2 Homework 1

**1.1** Prove that a loop $(L; \cdot, /, \backslash, 1)$ is abelian if and only if $\cdot$ is an abelian group operation

**1.2** Let $(R; +, 0, -, \cdot)$ be a commutative ring.
Recall that a congruence $\alpha$ is uniquely determined by the ideal $I_\alpha := 0/\alpha$.
Prove that $C(\alpha, \beta; 0)$ if and only if $I_\alpha \cdot I_\beta = 0$.

**1.3** Consider the loop $\underline{L}$ with universe $\mathbb{Z}_4 \times \mathbb{Z}_2$ given by the multiplication

$$(a,b)\cdot(c,d) = (a+c,\ b+d) \text{ unless } b=d=1$$

and $(a,1)\cdot(c,1) = (a*c,\ 0)$ where

| $*$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| 0 | 1 | 0 | 2 | 3 |
| 1 | 0 | 2 | 3 | 1 |
| 2 | 2 | 3 | 1 | 0 |
| 3 | 3 | 1 | 0 | 2 |

Consider the mapping $f: L \to \mathbb{Z}_2$ $(a,b) \mapsto b$, its kernel $\alpha$, and the $\alpha$-block $N$ of $1 = (0,0)$

- Prove that $f$ is a homomorphism
  ($N$ is so called normal subloop, it is abelian)
- Prove that $\alpha$ is not an abelian congruence
  (ie $C(\alpha, \alpha; 0)$ does not hold)