UA Exercises TCT

(1) Prove that an equivalence α is a congruence iff α is invariant under every unary polynomial

(2) Find all clones on $\{0,1\}$ containg the constant operations.

(3) $\underline{A}$ minimal algebra, $|A| \geq 3$, $f(x,\bar{y}) \in Pol(\underline{A})$. — 1st step —
If $\bar{c}, \bar{d} \in A$, then $x \mapsto f(x,\bar{c})$ is a permutation iff $x \mapsto f(x,\bar{d})$ is

deduce: $\underline{A}$ minimal, $|A| \geq 3$, $f(x,\bar{y}) \in Pol(\underline{A})$, $\bar{a},\bar{b} \in A^n$, $c,d \in A$

$$f(c,\bar{a}) = f(d,\bar{a}) \Rightarrow f(c,\bar{b}) = f(d,\bar{b})$$

(4) $\underline{A}$ algebra, U neigborhood with witness e

- $\underline{A}|_U = \{ef;\ f \text{ polynomial of } \underline{A}\}$
- $\underline{A}$ has Mal'cev/majority, then so does $\underline{A}|_U$
- $\underline{A}$ Mal'cev/majority, then $\operatorname{typ} \underline{A} \subseteq \{2,3\}$ / $\operatorname{typ} \underline{A} \subseteq \{3,4\}$

(5) $\underline{A}$ simple

- each minimal set is a neigborhood
- $a \neq b$, U minimal $\Rightarrow \exists f \in Pol_1(\underline{A})\ f(A) = U,\ f(a) \neq f(b)$
- $\forall a,b \in A\ \exists U_1, \dots, U_n$ minimal $a \in U_1,\ U_1 \cap U_2 \neq \emptyset,\ U_2 \cap U_3 \neq \emptyset, \dots\ b \in U_n$
- $\forall U,V$ minimal $\exists f,g \in Pol_1(\underline{A})\ f(U) = V,\ g(V) = U$
  $gf|_U = id|_U,\ fg|_V = id_V$