UAI
15.12.23
Practical 10

1) Let $\underline{A} = (\{0,1,2,3\}, *)$

$*$	0	1	2	3
0	1	2	1	0
1	0	3	2	3
2	1	0	1	0
3	2	3	2	1

Show there is no $f \in \mathrm{Clo}\,\underline{A}$ satisfying

a) $f(3,1,3,3,3) = 0$

b) $f(1,0,2,3,2) = 0$ and $f(1,0,0,3,2) = 1$.

2) Recall the definition of the dual
$f^d(x_1 \dots x_n) = \neg f(\neg x_1, \neg x_2 \dots, \neg x_n)$ for $f : \{0,1\}^n \to \{0,1\}$

a) Show that
$$\mathrm{Pol}(\{\neq\}) = \{ f \mid f^d = f \}$$

b) For any clone $\mathcal{C} = \mathrm{Pol}(\{R_1, \dots, R_k\})$ on $\{0,1\}$ find a nice relational description of
$$\mathcal{C}^d = \{ f^d \mid f \in \mathcal{C} \}.$$

3) For $n \geq 1$, let $OR_n = \{0,1\}^n \setminus \{(0,0 \dots 0)\}$.

a) Show that $\mathrm{Clo}((\{0,1\}, \to)) = \mathrm{Pol}(\{OR_n \mid n \geq 1\})$
($\underline{\text{Hint}}$: $f \in \mathrm{Clo}(\{0,1\}, \to) \Leftrightarrow \exists i : f(x_1 \dots x_n) \geq x_i$)

b) Prove that $\mathrm{Clo}(\{0,1\}, \to) \subsetneq \mathrm{Pol}(\{OR_1, \dots OR_k\})$ for any $k \geq 1$.

c) Conclude that $\mathrm{Clo}(\{0,1\}, \to) \neq \mathrm{Pol}(\{R_1 \dots, R_k\})$ for any finite set of relations.