UA I

8.12.23

Practical 9

1) Find explicit descriptions of the following clones:

a) $Clo(\{0,1\};$ no operations$)$

b) $Clo(\{0,1\}; \wedge)$

c) $Clo(\{0,1\}; +, 1)$

d) $Clo(\{0,1\}; d)$ with $d(x y z) = x - y + z$

e) $Clo(\{0,1\}; \neg)$

d) $Clo(\{0,1\}; \wedge, \neg)$

e) $Clo(\{0,1\}; NAND)$ with $NAND(x,y) = \neg(x \wedge y)$

How are they ordered w.r.t. $\leq$?

2) Show that $Clo(\{0,1\}; \wedge, \vee)$ is equal to the clone of monotone, idempotent operations on $\{0,1\}$.
What does this imply for free algebras $F_V(x_1 \ldots x_n)$ in $V = HSP(\{0,1\}; \wedge, \vee)$?

3) For an operation $f : \{0,1\}^n \to \{0,1\}$ let us define its dual f^d by $f^d(x_1, x_2 \ldots x_n) = \neg f(\neg x_1, \neg x_2, \ldots, \neg x_n)$
(e.g. $\wedge^d = \vee$).
Show that, for a clone $\mathcal{C}$ on $\{0,1\}$, also $\mathcal{C}^d := \{ f^d \mid f \in \mathcal{C} \}$ is a clone.

4) Let $\mathcal{C}$ be the set of all operations $f : \{0,1\}^n \to \{0,1\}$ such that $\exists i : f(x_1, x_2 \ldots x_n) \geq x_i$.

•) Show that $\mathcal{C}$ is a clone, and $Clo(\{0,1\}, \to) \leq \mathcal{C}$.

*) Prove that $Clo(\{0,1\}, \to) = \mathcal{C}$.

$\to$	0	1
0	1	1
1	0	1