UA I
1.12.23
Practicals 8

$\mathcal{V}, \mathcal{W}$ ... group varieties

$\Rightarrow \mathcal{V} \circ \mathcal{W} := \{ \underline{G} \text{ group} \mid \exists \underline{N} \trianglelefteq \underline{G},\ \underline{N} \in \mathcal{V},\ \underline{G}/\underline{N} \in \mathcal{W} \}$

is group variety

1) $\mathcal{A}$ ... variety of Abelian groups

$[x,y] := x^{-1}y^{-1}xy$

(i) Show that, for any group $\underline{G} = (G, \cdot, e, ^{-1})$

$$e/\lambda_{\mathcal{A}}^{\underline{G}} = Sg_{\underline{G}}(\{[x,y] : x, y \in G\})$$

(ii) Prove that $\mathcal{A} \cdot \mathcal{A}$ is axiomatised by the group laws together with $[[x,y],[z,w]] \approx e$.

2) $\mathcal{A}_n$ ... Abelian groups satisfying $x^n \approx e$.

(i) Show that

$$\mathcal{A}_3 \cdot \mathcal{A}_2 = \mathrm{Mod}(\text{group axioms} + \{x^6 \approx e, [x^2, y^2] \approx e, [x,y]^3 \approx e\})$$

(ii) $\mathcal{A}_2 \cdot \mathcal{A}_2 = \mathrm{Mod}(\text{group axioms} + \{(x^2y^2)^2 \approx e\})$

3) Cr_n ... commutative rings $(R, +, 0, -, \cdot)$ satisfying $x^n \approx x$.

$\mathbb{F}_9$ ... field of order 9.

Show that $HSP(\mathbb{F}_9)$ is equal to the subvariety of Cr_9 given by $3x (= x+x+x) \approx 0$.

Hint: SI elements of Cr_9 must be fields.