UA 1
24. 11.
Practicals 7

1) $\mathcal{V}$... variety of algebras $\underline{A} = (A, f)$
satisfying $f^6(x) \approx f^2(x)$.

Determine/draw $\underline{F}_V(\{x\})$ and $\underline{F}_V(\{x, y\})$

2) S... variety of semigroups
Show that $\underline{F}_S(X) \cong (\{\text{non-empty words over } X\}, \circ)$
concatenation
(e.g. $(x_4 x_1 x_3) \circ (x_3 x_{20}) = x_4 x_1 x_3 x_3 x_{20}$)

3) Let R be the variety of semigroups satisfying
$(x \cdot y) \cdot z \approx x \cdot z$, $x \cdot x \approx x$ (rectangular bands)

a) Describe $\underline{F}_R(X)$ for (finite) X.
b) Find a natural homomorphism $\underline{F}_S(X) \to \underline{F}_R(X)$.
c) Generalise (b) to free algebras of general varieties $W \subseteq V$.

4) Let V be the variety of distributive lattices.
a) describe $\underline{F}_V(\{x\})$, $\underline{F}_V(\{x, y\})$.
b*) try to find a finite upper bound on the size of $\underline{F}_V(\{x_1 \dots x_n\})$.

(In general it is undecidable to check if $F_V(X)$ is always finite for finite X. it is even open if $F_V(\{x, y\})$ is finite for $V = \text{groups} + (x^5 \approx 1)$ ... „Burnside problem")

5) V... variety of semilattices.
Show $\underline{F}_V(X) \cong (P(X) \setminus \{\emptyset\}, \cup)$