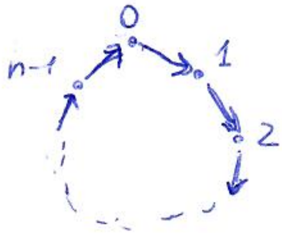
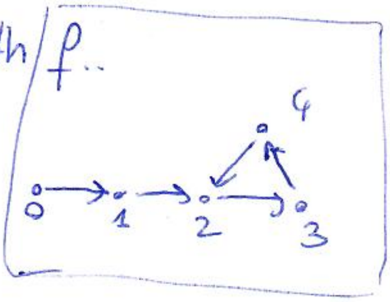
10.11.23
UAI
Practicals 6

1) For $n \geq 1$ let
$\underline{C}_n = (\{0, 1, \ldots, n-1\}, f(x))$
$f(x) = x + 1 \mod n$



For which $n \in \{2,3,4,5,6\}$ is $\underline{C}_n$
<u>simple</u> / <u>subdirectly irreducible</u> / <u>directly indecomposable</u>?

2) Let $\underline{A} = (\{0, 1, 2, 3, 4\}, f(x))$, with f..



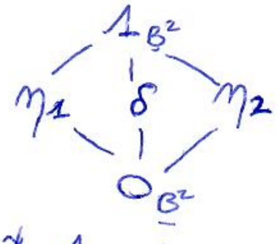
a) Draw $\mathrm{Con}\,\underline{A}$.
b) Is $\underline{A}$ <u>SI</u> / <u>directly indecomposable</u>?

3) Let $\underline{A} = (\{0,1\}, +, f(x))$ $\quad \underline{B} = (\{0,1\}, +, g(x))$ $\quad$ (+ is addition mod 2)
$f(x) = x$ $\quad g(x) = x + 1$.

a) Show $d : \underline{B}^2 \to \underline{A}$, $d(x,y) = x + y$ is surjective homomorphism.

b) Let $\delta = \ker(d)$
and $\eta_i = \ker(\pi_i)$ for $\pi_i : \underline{B}^2 \to B$ $\quad \pi_i(x_1, x_2) = x_i$

show $\mathrm{Con}\,\underline{B}^2 \cong$



with
$\eta_1 \circ \eta_2 = 1_{B^2}$
$\eta_1 \circ \delta = 1_{B^2}$
$\eta_2 \circ \delta = 1_{B^2}$

c) Prove that $\underline{B} \times \underline{B} \cong \underline{B} \times \underline{A} \not\cong \underline{A} \times \underline{A}$
Thus direct decompositions may not be unique.

4) Find $\underline{A}, \underline{B}$ such that

$\nexists\, h : \underline{A} \to \underline{A} \times \underline{B}$
$\nexists\, g : \underline{B} \to \underline{A} \times \underline{B}$ $\quad$ homom.

5) Represent the lattice $\underline{L}$

$\underline{L} =$  as a subdirect product of SI Lattices.