

1)  $\underline{A}, \underline{B}$  algebras of same type  
 $h: A \rightarrow B$ . Show that

$h: \underline{A} \rightarrow \underline{B}$  is homom.  $\Leftrightarrow \{(a, h(a)) \mid a \in A\} \subseteq \underline{A} \times \underline{B}$

2) Prove that the inverse of an isomorphism is an isomorphism.

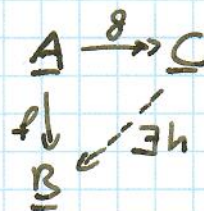
3)  $\nexists g, h: \underline{A} \rightarrow \underline{B}$  homom.,  $X \subseteq A$  with  $\text{Sg}_A(X) = A$   
Prove that  $g|_X = h|_X \Rightarrow g = h$

4) Find all homomorphisms  $h: (\mathbb{N}; +)^2 \rightarrow (\{1, -1\}, \cdot)$

5) (2nd isomorphism theorem)

a) Let  $f: \underline{A} \rightarrow \underline{B}$  homomorphisms, such that  $\ker(g) \subseteq \ker(f)$   
 $g: \underline{A} \rightarrow \underline{C}$  and  $g$  is surjective.

Prove  $\exists h: \underline{C} \rightarrow \underline{B}$  such that  $f = h \circ g$ .



b) Let  $h: \underline{A} \rightarrow \underline{B}$  homom.,  $\psi \in \text{Con}(\underline{B})$

Prove that there is an embedding

$$h/\psi: \underline{A}/h^{-1}(\psi) \rightarrow \underline{B}/\psi$$

c) Conclude that if  $\alpha, \beta \in \text{Con}(\underline{A})$   $\alpha \leq \beta$ , then

$$\exists \beta/\alpha \in \text{Con}(\underline{A}/\alpha): \underline{A}/\beta \cong (\underline{A}/\alpha)/(\beta/\alpha)$$

6) Find ~~the~~ classes of algebras witnessing that

$$PS \not\leq SP \quad PH \not\leq HP \quad SH \not\leq HS$$

7) In the situation of (3), assume that  $X$  is minimal generating set of  $\underline{A}$ .  
Does every map  $g: X \rightarrow B$  extend to a homomorphism  $g: \underline{A} \rightarrow \underline{B}$ ?