3.11
UA1
5th Pract.

1) $\underline{A}, \underline{B}$ algebras of same type, $h : A \to B$. Show that

$h : \underline{A} \to \underline{B}$ is homom. $\Leftrightarrow \{(a, h(a)) \mid a \in A\} \leq \underline{A} \times \underline{B}$

2) Prove that the inverse of an isomorphism is an isomorphism.

3) $g, h : \underline{A} \to \underline{B}$ homom., $X \subseteq A$ with $Sg_{\underline{A}}(X) = A$

Prove that $g\restriction_X = h\restriction_X \Rightarrow g = h$

4) Find all homomorphisms $h : (\mathbb{N}; +)^2 \to (\{1, -1\}, \cdot)$

5) (2nd isomorphism theorem)

a) Let $f : \underline{A} \to \underline{B}$, $g : \underline{A} \to \underline{C}$ homomorphisms, such that $\ker(g) \subseteq \ker(f)$ and g is surjective.

Prove $\exists h : \underline{C} \to \underline{B}$ such that $f = h \circ g$.

$$\begin{tikzcd} \underline{A} \arrow[r, two heads, "g"] \arrow[d, "f"'] & \underline{C} \arrow[dl, dashed, "\exists h"] \\ \underline{B} & \end{tikzcd}$$

b) Let $h : \underline{A} \to \underline{B}$ homom., $\psi \in \mathrm{Con}(\underline{B})$

Prove that there is an embedding

$h/\psi : \underline{A}/_{h^{-1}(\psi)} \to \underline{B}/\psi$

c) Conclude that if $\alpha, \beta \in \mathrm{Con}(\underline{A})$ $\alpha \leq \beta$, then

$\exists\, \beta/\alpha \in \mathrm{Con}(\underline{A}/\alpha) : \quad \underline{A}/_\beta \cong (\underline{A}/\alpha)/(\beta/\alpha)$

6) Find classes of algebras witnessing that

$PS \lneq SP \quad PH \lneq HP \quad SH \lneq HS$

7) In the situation of (3), assume that X is <u>minimal</u> generating set of $\underline{A}$.
Does every map $g : X \to B$ extend to a homomorphism $g : \underline{A} \to \underline{B}$?