27.10.23.
UAI
Practicals 4

1) $\underline{A} = (A, *)$ ... algebra of type (2)
$\theta \in Eq(A)$.
Show that $\theta \in Con(\underline{A}) \Leftrightarrow \forall a,b,c \in A$:
$$(a,b) \in \theta \Rightarrow \begin{cases} (a*c, b*c) \in \theta \\ (c*a, c*b) \in \theta \end{cases}$$

2) Let $\underline{A} = (\{0,1,2,3\}, *)$ with

*	0	1	2	3
0	0	2	1	1
1	2	1	0	2
2	1	0	2	0
3	1	2	0	3

Draw the lattices $Sub(\underline{A})$ and $Con(\underline{A})$

3) Let $\underline{A} = (\mathbb{Z}, +, \cdot) \times (\mathbb{Z}, \cdot, +)$. what is $Sg_{\underline{A}}(\{(1,0),(0,1)\})$?

4) Let $\underline{B} = (\{0,1\}, \wedge, \vee, \neg, 0, 1)$ be the 2-el. Boolean algebra.
Show that $(\mathcal{P}(X), \cap, \cup, {}^c, \emptyset, X) \cong \underline{B}^X$, for every set X.

5) $\underline{A}, \underline{B}$ ... algebras of same type
$h: \underline{A} \to \underline{B}$ homomorphism
(i) Let $U \leq \underline{A}$, $V \leq \underline{B}$. Is $h(U) \leq \underline{B}$ and $h^{-1}(V) \leq \underline{A}$?
(ii) Let $\theta \in Con(\underline{A})$, $\psi \in Con(\underline{B})$. Is $h(\theta) \in Con(\underline{B})$ and $h^{-1}(\psi) \in Con(\underline{A})$?
(here $h(\theta) = \{(h(x), h(y)) \mid (x,y) \in \theta\}$, $h^{-1}(\psi) = \{(x,y) \mid (h(x), h(y)) \in \psi\}$)
(iii) Let $X \subseteq A$. Is $h(Sg_{\underline{A}}^{A}(X)) = Sg_{\underline{B}}(h(X))$?

6) For $\underline{A} = (A, *)$, let us define the ~~~~ nucleus
$B = \{a \in A \mid \forall x,y \in A: (x*a)*y = x*(a*y)\}$.
Show that $B \leq \underline{A}$.
(*) Find $\underline{A}$ such that $B = \emptyset$.