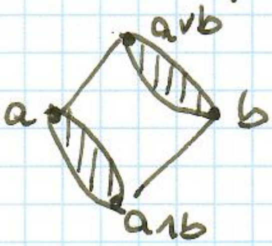
13.10.23
UAI
Practical 2

A lattice $\underline{L} = (L, \wedge, \vee)$ is called

distributive if $\forall x,y,z \in L$:	**modular** if $\forall a,b,x \in L$:
(i) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$	$a \leq b \Rightarrow$
(ii) $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$	$a \vee (x \wedge b) = (a \vee x) \wedge b$
e.g. $(P(A), \cap, \cup)$	e.g. subspaces of a vectorspace

1) Prove that every distributive lattice is modular.
2) a) Show that every $\underline{L}$ with $|L| \leq 4$ is distributive.
   b) Find two non-distributive lattices of size 5.
3) Show that (i) $\Leftrightarrow$ (ii).
4) Show that <u>every</u> lattice $\underline{L}$ satisfies
   - $x \vee (y \wedge z) \leq (x \vee y) \wedge (x \vee z)$
   - $a \leq b \Rightarrow a \vee (x \wedge b) \leq (a \vee x) \wedge b$

   (so distributivity/modularity are already determined by the reverse inequalities)
5) (<u>Diamond isomorphism theorem</u>)
   Let $\underline{L}$ be modular lattice and, $a, b \in L$.
   Prove that the intervals $I[a \wedge b, a]$ and $I[b, a \vee b]$ are isomorphic (as sublattices of $\underline{L}$).



6) A term $m(x,y,z)$ of an algebra $\underline{A}$ is called a <u>majority</u> if it satisfies $y \approx m(xyy) \approx m(yxy) \approx m(yyx)$.
   Show that lattices have majority terms.