

## Homework 5

Deadline 05.01.24 12:20

1. (10 points) Let  $\mathbf{L} = (\{0, 1, 2\}; \wedge, \vee)$  be the 3-element lattice. Show that  $\text{Clo}(\mathbf{L})$  is *not* the clone of all idempotent monotone operations on  $\{0, 1, 2\}$ .
2. (10 points) Let  $A$  be a set, and  $R = \{(x, y, z) \in A^3 \mid x = y \text{ or } y = z\}$ . Show that  $\text{Pol}(\{R\})$  is the clone of essentially unary operations (i.e. the clone of all operations of the form  $(x_1, \dots, x_n) \mapsto f(x_i)$ , for some  $1 \leq i \leq n$ ,  $f: A \rightarrow A$ ).
3. (10 points) In the lecture you saw that  $\mathcal{C} = \text{Pol}(\text{Inv}(\mathcal{C}))$  if  $\mathcal{C}$  is a clone on a *finite* set  $A$ . Prove that this is not true for infinite  $A$  (Hint: consider the clone generated by all bijections  $A \rightarrow A$ ).