NMAG 405 - Universal Algebra 1 - fall semester 2020/21

Homework 5

Deadline 07.01.2021, 9:00

1. (10 points) Let $\mathcal{C} = \mathrm{Clo}(\mathbf{A})$, where $\mathbf{A} = (\{1,2,3,4\}, *)$ with

$*$	1	2	3	4
1	2	3	2	1
2	1	4	3	4
3	2	1	2	1
4	3	4	3	2

   (a) Prove that there is no 5-ary operation $f \in \mathcal{C}$ satisfying $f(4,2,4,4,4) = 1$

   (b) Prove that there is no 5-ary operation $f \in \mathcal{C}$ satisfying $f(2,1,3,4,3) = 1$ and $f(2,1,1,4,3) = 2$

   (Hint: invariant relations)

2. (10 points) Let $\mathbf{L} = (\{0,1,2\}, \wedge, \vee)$ be the three-element lattice. Find a monotone idempotent operation that is not in $\mathrm{Clo}(\mathbf{L})$.

3. (10 points) In the lecture you saw that $\mathcal{C} = \mathrm{Pol}(\mathrm{Inv}(\mathcal{C}))$, if $\mathcal{C}$ is a clone on some finite set A. Show that for clones on an infinite A this is not true in general (Hint: study the clone generated by all bijections $A \to A$).