NMAG 405 - Universal Algebra 1 - fall semester 2019/20

Homework 2

Deadline 19.11.2019, 11:30

1. (10 points) Determine all the subalgebras and congruences of $(\mathbb{N}, *)$ where $x * y = \max(x, y) + 1$. Draw the lattices Sub and Con.

2. (10 points) Let $\mathbf{G} = (G, \cdot, ^{-1}, e)$ be a group. Prove that there is a lattice isomorphism between the lattice of normal subgroups of $\mathbf{G}$ and the lattice of congruences of $\mathbf{G}$.

3. (10 points) For a fixed prime p consider the algebra $\mathbf{A} = (\{0, 1, \ldots, p-1\}, m)$, where m is a ternary operation defined by $m(x, y, z) = x - y + z \mod p$. Prove that for any n, R is a subuniverse of $\mathbf{A}^n$ if and only if R is empty or an affine subspace of $\mathbb{Z}_p^n$. (Recall from linear algebra that R is an affine subspace iff it is closed under all affine combinations.)