NMAG 405 Universal Algebra 1, fall semester 2017–2018

Homework 5

Deadline 8 Jan 2018, 10:40

5.1. (10 points) Let $\mathbf{L} = (\{0,1,2\}; \wedge, \vee)$ be the lattice corresponding to the three–element chain $0 < 1 < 2$. Find a monotone idempotent operation which is not in $\mathrm{Clo}(\mathbf{L})$. (Hint: calculate the binary part.)

5.2. (10 points) Consider $\mathcal{C} = \mathrm{Clo}(\mathbf{A})$, where $\mathbf{A} = (\{1,2,3,4\}; *)$ is given by

$*$	1	2	3	4
1	2	3	2	1
2	1	4	3	4
3	2	1	2	1
4	3	4	3	2

(a) Prove that no 5-ary operation f satisfying $f(2,4,2,2,4) = 1$ is in $\mathcal{C}$.

(b) Prove that no 5-ary operation f satisfying $f(2,1,3,4,3) = 1$, $f(2,1,1,4,3) = 2$ is in $\mathcal{C}$.

(Hint: use natural compatible relations of arity 1 and 2.)

5.3. (10 points) Let $\mathbb{A} = (\{0,1\}; R_{000}, R_{001}, R_{011}, R_{111})$, where $R_{abc} = \{0,1\}^3 \setminus \{(a,b,c)\}$.

(a) Pp-define each ternary relation from $\mathbb{A}$.

(b) Pp-define each unary and binary relation from $\mathbb{A}$.

(c) Pp-define the 4-ary relations $R_{abcd} = \{0,1\}^4 \setminus \{(a,b,c,d)\}$ from $\mathbb{A}$. (The binary relation $x = \neg y$ may help)

(d) Pp-define every relation from $\mathbb{A}$.