Universal Algebra Exercises - Homework 4

Exercise 1. Let R be ring with unity and let RMod be the variety of R-modules. Show that the free algebra $F_{R\text{Mod}}(X)$ is given by

$$\bigoplus_{x\in X} R = \left\{(r_x)_{x\in X} \mid r_x = 0 \text{ for all but finitely many } x \in X\right\}$$

with component-wise operations.

Exercise 2. Let $\mathbb{L}$ and $\mathbb{M}$ be two bounded lattices with more than one element. Denote by $\mathbb{L} \oplus \mathbb{M}$ the poset on the set $L \amalg M$ where

$$x \leq y \iff \begin{cases} x \leq y \text{ in } \mathbb{L} \text{ or} \\ x \leq y \text{ in } \mathbb{M} \text{ or} \\ x \in L \text{ and } y \in M. \end{cases}$$

Show that $\mathbb{L} \oplus \mathbb{M}$ is a lattice and that $\mathrm{HSP}(\mathbb{L}, \mathbb{M}) = \mathrm{HSP}(\mathbb{L} \oplus \mathbb{M})$.

Exercise 3. Let X be a finite set. Show that the clone of all operations on X is generated by all binary operations on X.