CSPs over Finite Structures

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$\mathrm{CSP}(\mathbb{A})$ where $\mathbb{A} = (A;$ relations, operations$)$

INPUT: $\mathbb{X}$ of the same signature

QUESTION: $\exists$ homo $\mathbb{X} \to \mathbb{A}$?

- complexity known for CSP(relations)
  [Bulatov, Zhuk]
- CSP(operations)
  - contains all CSPs(relations)
    [Feder, Madelaine, Stewart]
  - richer [this work]
- CSP(relations, operations)
  is a LHS restriction of CSP(relations)

Boolean dichotomy

Our results

Thm complexity depends on
- Clo(operations)
- Partial Pol(relations)

Thm Algebra $\underline{A}$ satisfies nontrivial idempotent identities
$\Rightarrow |\mathrm{Hom}(\underline{X}, \underline{A})| \le \mathrm{poly}(|\underline{X}|)$

Thm For some $\underline{A}$ TFAE
- $\mathrm{CSP}(\underline{A}$ + any relations$)$ is in P
- $|\mathrm{Hom}(\underline{X}, \underline{A})| \le \mathrm{poly}(|X|)$
  ↳ + can be enumerated in P
- no subalgebra of $\underline{A}$ has a nontrivial strongly abelian congruence

News For all $\underline{A}$