Využití hybridní metody vícekriteriálního rozhodování za nejistoty

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Outline

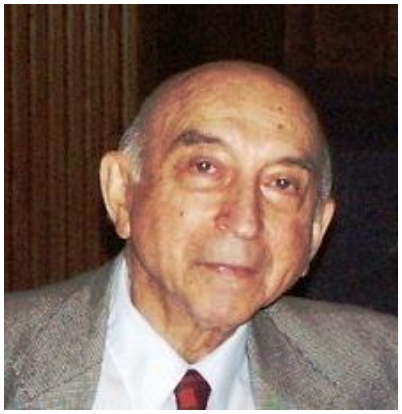
- Multiple criteria decision making
- Classification of MCDM methods
- TOPSIS method
- Fuzzy extension of MCDM method
- Application

Multiple criteria decision making

Milan Zelený: Every decision-making process must have at least two different, conflicting criteria. If there is only a single criterion (an objective, purpose), then the “measurement plus search” procedure will do; this is not decision making. To decide means to deal with "tradeoffs" (among criteria). Decision-making and optimization with single criterion is clearly an oxymoron.

Fathers of MCDM theory

Lofti Zadeh (1921-2017)



Bernard Roy (1934-2017)

Thomas L. Saaty (1926-2017)

Classification of discrete MCDM methods

Preference information on alternatives					
aspiration levels	ordinal information	cardinal information			
		utility function	distance of alternative from bazal or ideal	preference relation	marginal rate of substitution
Methods					
PRIAM	Lexicographic ORESTE Permutation	WSA	TOPSIS PROMETHEE ELECTRE	AHP	Compensation

Source: J. Jablonský, Operační výzkum (2002)

Notation

- n alternatives: $a_i, \quad i = 1, \ldots, n$
- k criteria functions $K_j, \; j = 1, \ldots, k$
- Values of K_j in a_i are denoted by y_{ij} and we call them criteria values.
- $\mathbf{Y} = (y_{ij})_{n \times k}$ is called criteria matrix
- We assume that all criteria are maximized and all the criteria values are positive,

$$y_{ij} > 0, i = 1, \ldots, n, j = 1, \ldots, k$$

TOPSIS (Technique for Order Preference by Similarity to Ideal Solution)

TOPSIS method was introduced in the 1980s by Hwang and Yoon. It consists of following steps:

1. normalization of the criteria matrix
2. construction of weighted criteria matrix
3. identification of ideal and basal solution
4. computing distance of alternatives from ideal and basal solution
5. computation of similarity of alternatives to the ideal solution

TOPSIS, step 1: normalization of the criteria matrix

Construction of normalized matrix

$$\mathbf{R} = (r_{ij})_{n \times k}$$

using simplified formula of Garcia-Cascalez

$$r_{ij} = \frac{y_{ij}}{y_j^{max}}, i = 1,\ldots,n,\ j = 1,\ldots,k,$$

where $y_j^{max} = \max_{i = 1,\ldots,n} y_{ij}$.

TOPSIS, step 2: construction of weighted matrix

Computation of weighted criteria matrix

$\mathbf{Z} = (z_{ij})_{n \times k}$ using normalized matrix $\mathbf{R}$ and the weights of criteria

$w_j, j = 1, \ldots, k,$

in a way:

$$z_{ij} = r_{ij} \cdot w_j, \; i = 1, \ldots, n, \; j = 1, \ldots, k.$$

TOPSIS, steps 3-4: distance of alternatives from ideal and basal solution

We define ideal solution $A^+ = (A_1^+, \ldots, A_k^+)$

and basal solution $A^- = (A_1^-, \ldots, A_k^-)$ as

$$A_j^+ = \max_{i=1,\ldots,n} z_{ij}, A_j^- = \min_{i=1,\ldots,n} z_{ij}, j = 1, \ldots, k,$$

Then we compute distance of individual alternatives $a_i,\ i = 1, \ldots, n,$ from ideal and basal solution respectively:

$$d_i^+ = \sqrt{\sum_{j=1}^{k} (z_{ij} - A_j^+)^2}\ , \qquad d_i^- = \sqrt{\sum_{j=1}^{k} (z_{ij} - A_j^-)^2}\ ,$$

TOPSIS, step 5: computation of similarity of alternatives to the ideal solution

The index $c_i = \frac{d_i^-}{d_i^+ + d_i^-}, \ i = 1, \ldots, n,$ is computed for every alternative.

Alternatives $a_i, i = 1, \ldots, n,$ are ranked according to the values c_i. The alternative a_i with maximal value of c_i is selected as the compromise alternative a^*. It is the one that is "as far from basal and, at the same time, as close to ideal as possible".

Fuzzy extension of TOPSIS

- *Definition:* Let U be the set called universe of discourse. Then the fuzzy set $\tilde{A}$ on universe U is defined by a mapping $\mu_{\tilde{A}}: U \to \langle 0, 1 \rangle$, called membership function of the fuzzy set $\tilde{A}$. For every $x \in U$, the value $\mu_{\tilde{A}}(x)$ determines the grade of membership of x to fuzzy set $\tilde{A}$. The set of all fuzzy sets on universe U is denoted by $\mathcal{F}(U)$.

Triangular fuzzy numbers

Triangular fuzzy number can be written as $\widetilde{C} = (c_l, c_m, c_u)$, where its membership function is:

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - c_l}{c_m - c_l}, & x \in \langle c_l, c_m) \\ \dfrac{c_u - x}{c_u - c_m}, & x \in \langle c_m, c_u \rangle \\ 0, & \text{elsewhere} \end{cases}$$

Real numbers c_l, c_m, c_u represent the smallest likely value, the most probable value and the largest possible value of a corresponding fuzzy event respectively.

Standard fuzzy arithmetic

Let $\widetilde{C} = (c_l, c_m, c_u)$, $\widetilde{D} = (d_l, d_m, d_u)$ be positive triangular fuzzy numbers. We define:

$\widetilde{C} + \widetilde{D} = (c_l + d_l, c_m + d_m, c_u + d_u)$,

$\widetilde{C} \cdot \widetilde{D} = (c_l \cdot d_l, c_m \cdot d_m, c_u \cdot d_u)$,

$\frac{\widetilde{C}}{\widetilde{D}} = \left(\frac{c_l}{d_u}, \frac{c_m}{d_m}, \frac{c_u}{d_l}\right)$

Distance between fuzzy numbers $\widetilde{C}$, $\widetilde{D}$ is defined by the formula

$$d\left(\widetilde{C}, \widetilde{D}\right) = \sqrt{\frac{1}{3}\left[(c_l - d_l)^2 + (c_m - d_m)^2 + (c_u - d_u)^2\right]}.$$

Input of fuzzy TOPSIS

The input data consists of fuzzy weights of criteria $\widetilde{w_j} = \left(w_{jl}, w_{jm}, w_{ju}\right), j = 1, \ldots, k,$

and the fuzzy criteria matrix $\widetilde{\mathbf{Y}} = (\tilde{y}_{ij})_{n \times k}.$

Its elements are triangular fuzzy numbers $\tilde{y}_{ij}$ with significant values $(y_{ijl}, y_{ijm}, y_{iju}),$

Steps of the fuzzy TOPSIS

1. $\tilde{r}_{ij} = \left(\frac{y_{ijl}}{y_j^{max}}, \frac{y_{ijm}}{y_j^{max}}, \frac{y_{iju}}{y_j^{max}} \right),\ i = 1, \ldots, n,$
   $j = 1, \ldots, k,$ where $y_j^{max} = \max_{i = 1,\ldots,n} y_{iju}$
2. $\tilde{z}_{ij} = \tilde{r}_{ij} \cdot \tilde{w}_j,\ i = 1, \ldots, n,\ j = 1, \ldots, k$
3. $\tilde{A}_j^+ = (1,1,1),\ \tilde{A}_j^- = (0,0,0), j = 1, \ldots, k,$
4. $d_i^+ = \sum_{j=1}^k d(\tilde{z}_{ij}, \tilde{A}_j^+)\ , d_i^- = \sum_{j=1}^k d(\tilde{z}_{ij}, \tilde{A}_j^-)\ ,$
   $i = 1, \ldots, n.$
5. $c_i = \frac{d_i^-}{d_i^+ + d_i^-}$

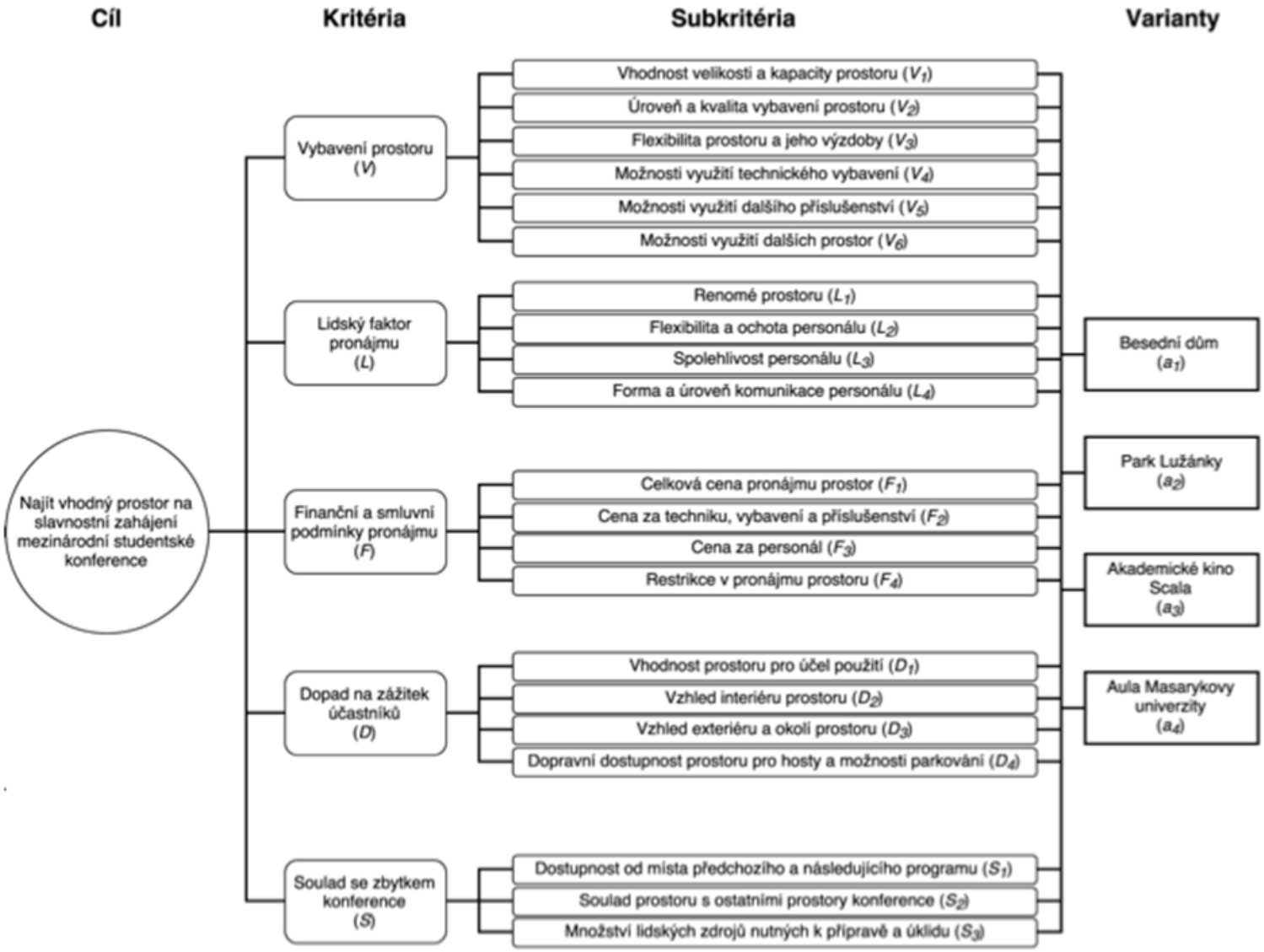
Aplikace na rozhodovací problém

Výběr prostor pro slavnostní zahájení 85. mezinárodního zasedání Evropského parlamentu mládeže - Brno 2017 http://eyp.cz/is17brno/

Varianty představovaly čtyři brněnské prostory:

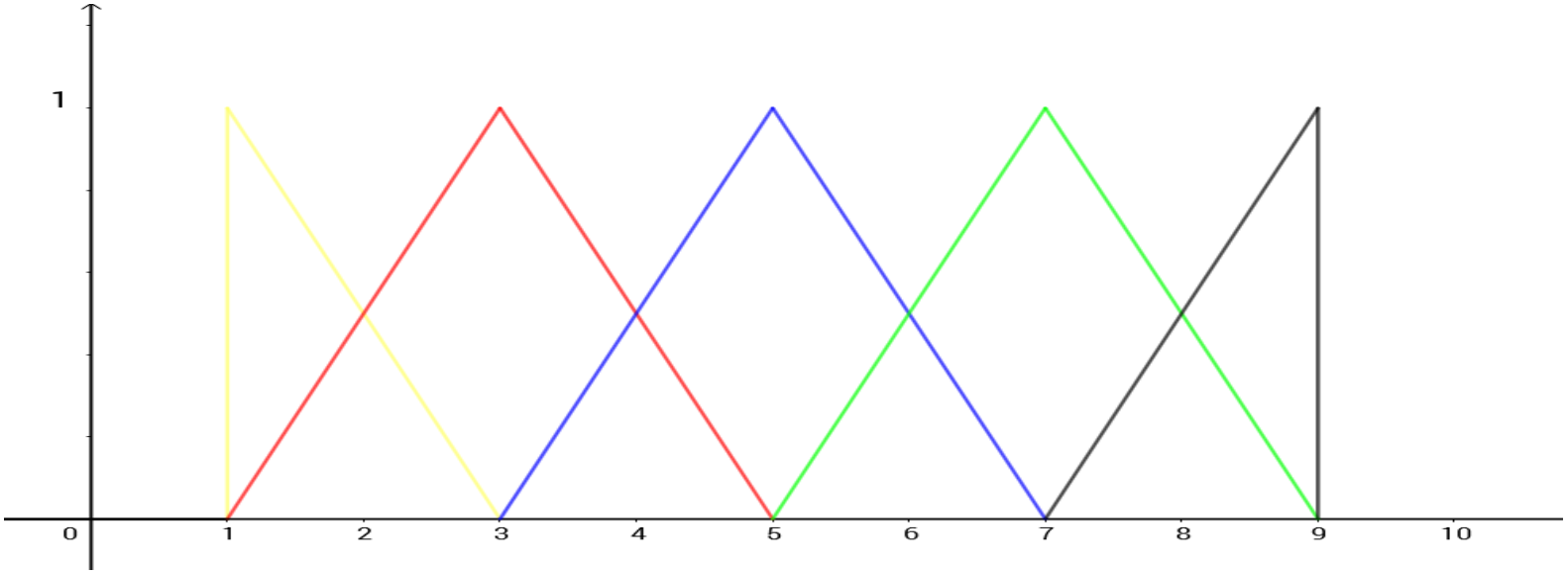
- Besední dům (a_1),
- park Lužánky (a_2),
- Akademické kino Scala (a_3),
- Aula Masarykovy univerzity (a_4)

Struktura rozhodovacího problému



Stupnice fuzzy hodnot pro hodnocení variant

Fuzzy číslo	Lingvistický termín
$\widetilde{1} = (1,1,3)$	varianta naplňuje subkritérium velmi málo
$\widetilde{3} = (1,3,5)$	varianta naplňuje subkritérium málo
$\widetilde{5} = (3,5,7)$	varianta naplňuje subkritérium středně
$\widetilde{7} = (5,7,9)$	varianta naplňuje subkritérium dobře
$\widetilde{9} = (7,9,9)$	varianta naplňuje subkritérium velmi dobře



Kriteriální matice a výsledný index

$$
\begin{array}{c}
 & V_1 & V_2 & V_3 & V_4 & V_5 & V_6 & L_1 & L_2 & L_3 & L_4 & F_1 & F_2 & F_3 & F_4 & D_1 & D_2 & D_3 & D_4 & S_1 & S_2 & S_3 \\
a_1 & \left(\right. \widetilde{9} & \widetilde{9} & \widetilde{3} & \widetilde{9} & \widetilde{7} & \widetilde{9} & \widetilde{9} & \widetilde{9} & \widetilde{9} & \widetilde{9} & \widetilde{9} & \widetilde{9} & \widetilde{7} & \widetilde{9} & \widetilde{9} & \widetilde{9} & \widetilde{7} & \widetilde{9} & \widetilde{5} & \widetilde{7} & \widetilde{7} \\
a_2 & \widetilde{9} & \widetilde{3} & \widetilde{9} & \widetilde{5} & \widetilde{7} & \widetilde{7} & \widetilde{5} & \widetilde{7} & \widetilde{7} & \widetilde{3} & \widetilde{7} & \widetilde{7} & \widetilde{3} & \widetilde{9} & \widetilde{7} & \widetilde{3} & \widetilde{9} & \widetilde{3} & \widetilde{5} & \widetilde{7} & \widetilde{3} \\
a_3 & \widetilde{9} & \widetilde{7} & \widetilde{5} & \widetilde{9} & \widetilde{7} & \widetilde{9} & \widetilde{7} & \widetilde{9} & \widetilde{9} & \widetilde{9} & \widetilde{9} & \widetilde{7} & \widetilde{7} & \widetilde{5} & \widetilde{7} & \widetilde{5} & \widetilde{7} & \widetilde{9} & \widetilde{7} & \widetilde{7} & \widetilde{3} \\
a_4 & \widetilde{3} & \widetilde{7} & \widetilde{3} & \widetilde{7} & \widetilde{5} & \widetilde{9} & \widetilde{7} & \widetilde{3} & \widetilde{7} & \widetilde{9} & \widetilde{5} & \widetilde{9} & \widetilde{9} & \widetilde{3} & \widetilde{9} & \widetilde{9} & \widetilde{7} & \widetilde{9} & \widetilde{9} & \widetilde{7} & \widetilde{5} \left. \right)
\end{array}
$$

Alternativa	d_i^+	d_i^-	c_i
a_1	1,372841	1,2127350	0,4690387
a_2	1,501998	0,9879444	0,3967740
a_3	1,360780	1,2210642	0,4729426
a_4	1,398845	1,1658255	0,4545713

Výsledné pořadí

1. 

2. 

3. 

4. 

Reference

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Děkuji za pozornost !