Joint estimation of parameters of mortgage portfolio

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Outline

- Motivation
- Our problem
- Numerical algorithm
- Work in progress

Motivation

- **WHAT:** A dynamic multifactor model
- **WHERE:** Mainly in banks
- **ABILITY:** To evaluate credit risk
- **DATA:** Mortgage loans
- **WHY:** Mortgage crisis

Setting

- a portfolio of N loans
- amount of loans are unit
- annuity instalment of loan agreement
- each loan is secularized by a collateral value
- default happens if wealth of client decrease under certain threshold
- the wealth is driven by common factor Y_t and individual factor Z_t^i
- the collateral value is driven by common factor I_t and individual factor E_t^i

Notation

- the overall default rate (PD)
  - $Q_t = \lim_{N \to \infty} \frac{\sum_{1<i<N} Q_t^i}{N}; \quad Q_t^N = \frac{\sum_{1 \leq i \leq N} D_t^i}{N_t}, t > 0$
- loss given default (LGD) of the creditor at time t
  - $G_t = \lim_{N \to \infty} \frac{\sum_{1<i<N} G_t^i}{\sum_{1<i<N} D_i}; \quad G_t^i = \frac{D_t^i \max(0, h_t - P_t^i)}{h_t}$
- where
  - N_t is number of debts at time t
  - D_t is number of defaults at time t
  - P_t is the collateral value

Our problem

- We observe $G_1, \ldots, G_n,\ Q_1, \ldots, Q_n$ but not factors $I_1, Y_1, \ldots, I_n, Y_n$
- We want to predict G's and Q's
- Solution
  - Transformation G's and Q's into I's and Y's
  - Then predict I's and Y's (by for example VECM with unknown parameters)
  - And finally transform I's and Y's back into G's and Q's
- In addition we need to estimate parameters of both common and individual factor

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The Transformation

The goal is to find transformation function ψ and its inversion

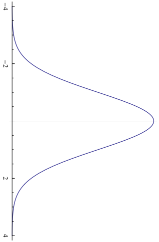
- $(Y_1, I_1, \ldots, Y_t, I_t) \overset{\psi}{\rightsquigarrow} (Q_t, G_t)$

Theorem

- *ψ is strictly decreasing*
- *ψ is continuously differentiable*
- *ψ is bijection between $\mathbb{R}$ and $(0, 1)$*
- *Inverse of ψ exists and is continuously differentiable*

- Ψ is intractable

Numerical Technique - Illustration



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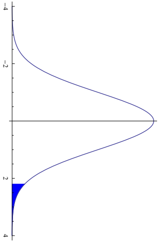

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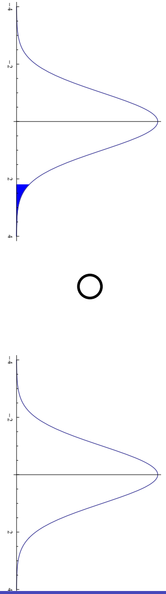

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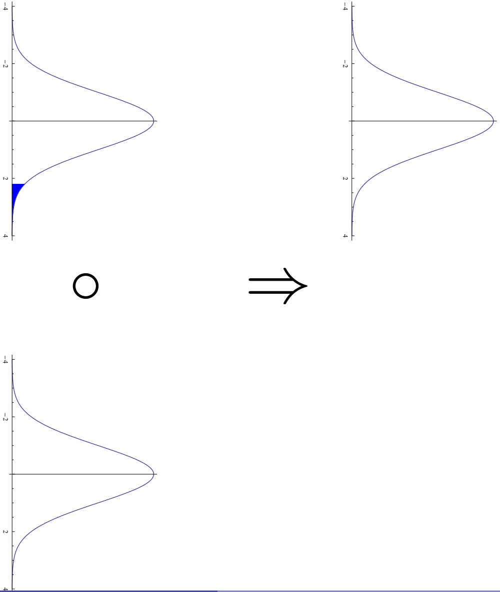
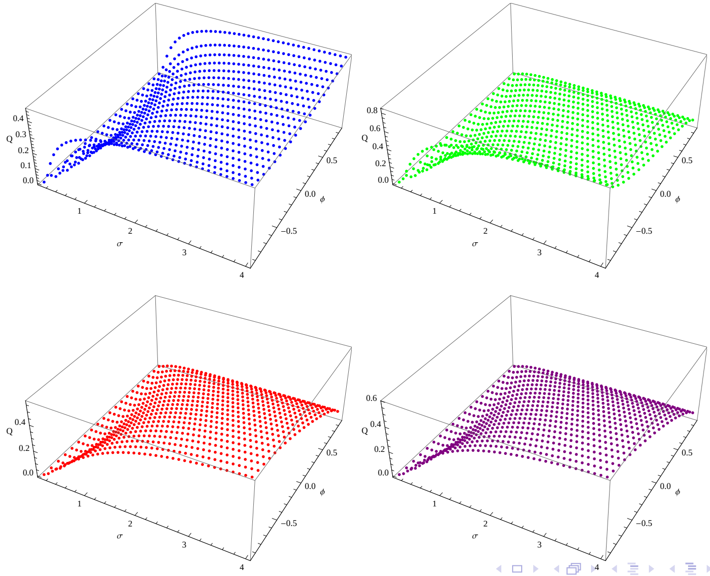


Numerical ilustration

- U_1, U_2 are normal
- $Y^* = 1, t \geq 1$
- $\sigma_1^2 = \frac{\sigma^2}{1-\phi^2}$ which corresponds to stationary Z

- Q_1, Q_2, Q_3, Q_4 are given that $Y_1^* = Y_2^* = Y_3^* = Y_4^* = 1$ and normal stationary Z for various ϕ and σ



MLE

- The problem: We observe Q's and G's and known dynamics of Y, I
- $(G_1, Q_1, \ldots, G_{t-1}, Q_{t-1}) \overset{\psi^{-1}}{\rightsquigarrow} (Y_1, I_1, \ldots Y_{t-1}, I_{t-1}) \overset{VECM}{\longrightarrow} (Y_1, I_1, \ldots Y_t, I_t) \overset{\psi}{\rightsquigarrow} (G_1, Q_1, \ldots, G_t, Q_t)$
- The log-likelihood function of (Q, G)
  $l(y, \theta, \sigma) = \sum_{i=1}^{n} \left(\log f_i^Y(\psi^{-1}(y_i; \sigma); \theta) + \log \left| \frac{1}{\psi'(\psi^{-1}(y_i; \sigma); \sigma)} \right| \right)$
  - f^Y is density of common factor Y
  - θ parameters of density f^Y
  - σ variance of initial wealth
- work in progress - asymptotics (consistence and normality)

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Conclusion

- We define a multi-periodic model of mortgage portfolio
- We propose a numerical technique for transformation form Y, I to G, Q
- in progress
  - estimation of parameters of both common and individual factor
  - aszmptotics of transformed likelihood function

Thank you for attention.