Reconstructing the topology of the polymorphism clone of the random graph

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joint work with Manuel Bodirsky and Michael Pinsker

Prague, 2014

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Theorem [G. Ahlbrandt, M. Ziegler]
If Δ is ω-categorical, then Δ and Γ are first-order bi-interpretable iff $\mathrm{Aut}(\Delta) \cong \mathrm{Aut}(\Gamma)$ as topological groups.

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Fact: It is consistent with ZF that for every ω-categorical structure Δ the topological group Aut(Δ) has automatic homeomorphicity.

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From groups to monoids

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Proposition

Let $\mathcal{M}$ and $\mathcal{M}'$ be closed submonoids of $\mathcal{O}^{(1)}$ with dense subsets of invertibles $\mathcal{G}$ and $\mathcal{G}'$. Let $\xi : \mathcal{G} \to \mathcal{G}'$ be a continuous isomorphism. Then ξ extends to an isomorphism $\bar{\xi} : \mathcal{M} \to \mathcal{M}'$ which is a homeomorphism.

From groups to monoids

Proposition (M. Bodirsky, M. Pinsker, AP '13)

Let $\mathcal{M}$ be a closed submonoid of $\mathcal{O}^{(1)}$ whose group of invertible elements $\mathcal{G}$ is dense in $\mathcal{M}$ and has automatic homeomorphicity. Assume that the only injective endomorphism of $\mathcal{M}$ that fixes every element of $\mathcal{G}$ is the identity function $\mathrm{id}_{\mathcal{M}}$ on $\mathcal{M}$. Then $\mathcal{M}$ has automatic homeomorphicity.

Automatic homeomorphicity of monoids

Theorem (M. Bodirsky, M. Pinsker, AP '13)

Let Δ be a countable homogeneous relational structure such that $\mathrm{Aut}(\Delta)$ has no algebraicity and with the joint extension property such that $\mathrm{Aut}(\Delta)$ has automatic homeomorphicity. Then the monoid $\overline{\mathrm{Aut}(\Delta)}$ of self-embeddings of Δ has automatic homeomorphicity.

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Non-examples: $(\mathbb{Q}, <)$, Henson graphs

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there exist unary $(\alpha^i)_{i\in\mathbb{N}}$ and $(\beta_1^i)_{i\in\mathbb{N}}, \ldots, (\beta_n^i)_{i\in\mathbb{N}}$ in $\mathcal{C}$ with

- $g^i(x_1, \ldots, x_n) = \alpha^i(f_U(\beta^i(x_1), \ldots, \beta^i(x_n)))$ and
- $(\alpha^i)_{i\in\mathbb{N}}$ and $(\beta_1^i)_{i\in\mathbb{N}}, \ldots, (\beta_n^i)_{i\in\mathbb{N}}$ converge.

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Corollary

$\mathcal{H}$ has automatic homeomorphicity.

Automatic homeomorphicity of $\mathrm{Pol}(V, E)$

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Cheating!

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Openness follows from TB + CH.

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Problem 2 Does End(H_n, E) (or Pol(H_n, E)) have automatic homeomorphicity for (any) $n \geq 3$?