On the complexity of testing for Jónsson terms

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Congruence Distributivity

Theorem (Jónsson)

$\mathcal{V}$ is congruence distributive if and only if for some $k > 1$ it has terms $p_i(x, y, z)$ for $0 \leq i \leq k$ which satisfy the identities:

$$\begin{aligned} p_0(x, y, z) &\approx x \\ p_i(x, y, x) &\approx x \text{ for all } i \\ p_i(x, x, y) &\approx p_{i+1}(x, x, y) \text{ for all } i \text{ even} \\ p_i(x, y, y) &\approx p_{i+1}(x, y, y) \text{ for all } i \text{ odd} \\ p_k(x, y, z) &\approx z \end{aligned}$$

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p_k(x, y, z) &\approx z
\end{aligned}$$

Definition

For $k > 1$, an algebra $\mathbf{A}$ is said to be a CD(k) algebra if it has a sequence of term operations $p_i(x, y, z)$, $0 \leq i \leq k$, that satisfy the equations in the above theorem.

Testing for Jónsson terms

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Remark

There is a straightforward, but inefficient algorithm to settle this question: Compute the free algebra in $\mathbf{V}(\mathbf{A})$ *generated by* $\{x, y, z\}$ *and look for a sequence of Jónsson terms of length* $k+1$.

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Let $\mathbf{A}$ be a finite algebra. The following decision problems are EXP-TIME complete:

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Theorem (Freese-Val., Horowitz)

Let $\mathbf{A}$ *be a finite algebra. The following decision problems are EXP-TIME complete:*

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Theorem (Freese-Val., Horowitz)

Let $\mathbf{A}$ be a finite algebra. The following decision problems are EXP-TIME complete:

- *Is $\mathbf{A}$ is CD(k) for some $k > 1$?*
- *For a fixed $k > 2$, is $\mathbf{A}$ CD(k)?*

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Fact

Płonka showed that the clone of idempotent operations of a group $(G, \cdot, ^{-1}, e)$ *is generated by the two idempotent ternary operations* $x^{-1}yz$ *and* xyz^{-1}.

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Take an algebra of minimal size in $\mathbf{V}(\mathbf{A})$ whose congruence lattice is not distributive and hammer away at it until it can be shown to lie in $\mathbf{HS}(\mathbf{A}^3)$.

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Take an algebra of minimal size in $\mathbf{V}(\mathbf{A})$ whose congruence lattice is not distributive and hammer away at it until it can be shown to lie in $\mathbf{HS}(\mathbf{A}^3)$. Our algorithm amounts to searching through the 3-generated subalgebras of $\mathbf{A}^3$, looking for a failure of congruence distributivity.

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- To determine if **A** has a majority term one needs to find a term operation $m(x, y, z)$ of **A** such that for all 6-tuples $\sigma = (a_1, a_2, a_3, b_1, b_2, b_3) \in A^6$,

$$m(a_1, a_1, b_1) = a_1, \quad m(a_2, b_2, a_2) = a_2, \text{ and } \quad m(b_3, a_3, a_3) = a_3.$$

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- Independently, Horowitz and McKenzie observed that **A** will have a majority term if and only if for every 6-tuple σ as above, **A** has a term operation $m^\sigma(x, y, z)$ that satisfies the displayed equalities.

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- Independently, Horowitz and McKenzie observed that $\mathbf{A}$ will have a majority term if and only if for every 6-tuple σ as above, $\mathbf{A}$ has a term operation $m^{\sigma}(x, y, z)$ that satisfies the displayed equalities.
- Thus, if $\mathbf{A}$ has enough "local" majority term operations, then it will have a majority term operation.

Testing for local term operations

Proposition

For **A** *an algebra and* $\sigma = (a_1, a_2, a_3, b_1, b_2, b_3) \in A^6$, **A** *will have a term operation* m^σ *satisfying*

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if and only if the tuple (a_1, a_2, a_3) *is in the subuniverse of* $\mathbf{A}^3$ *generated by*

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Remark

It follows that to test that a finite idempotent algebra $\mathbf{A}$ *has a majority term, one need only search through* A^6*, checking that the above subuniverse condition holds.*

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It follows that to test that a finite idempotent algebra $\mathbf{A}$ *has a majority term, one need only search through* A^6*, checking that the above subuniverse condition holds.*
This can be done using a poly-time algorithm, since this sort of subuniverse generation can be performed quickly.

Local Jónsson terms

Definition

Let $k > 1$ and $\mathbf{A}$ be a finite idempotent algebra and let

$$\sigma = (a_1, a_2, \dots, a_k, b_1, b_2, \dots, b_k, c_1, c_2, \dots, c_{k-1}, d_1, \dots, d_{k-1})$$

be a tuple from A^{4k-2}. The sequence of term operations $p_i^\sigma(x, y, z)$, $1 \leq i < k$ is a local Jónsson sequence for σ if

$$\begin{aligned} p_i^\sigma(c_i, d_i, c_i) &= c_i \quad \text{for all } i\text{, and} \\ a_1 &= p_1^\sigma(a_1, a_1, b_1), \\ p_1^\sigma(a_2, b_2, b_2) &= p_2^\sigma(a_2, b_2, b_2), \\ p_2^\sigma(a_3, a_3, b_3) &= p_3^\sigma(a_3, a_3, b_3), \\ &\vdots \end{aligned}$$

CD(k) for $k > 2$

Theorem (Kazda-Val.)

Let $\mathbf{A}$ *be a finite idempotent algebra and* $k > 1$. $\mathbf{A}$ *is a CD*(k) *algebra if and only if for every* $4k - 2$ *tuple* σ *from* A, $\mathbf{A}$ *has a local Jónsson sequence for* σ.

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Corollary

For $k > 1$*, there is a poly-time algorithm to determine if a given finite idempotent algebra is CD*(k)*.*

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- *The proof of the theorem is in the Krakow-Prague style and uses potatoes and coloured graphs.*
- *The corollary follows, since checking for local Jónsson sequences amounts to computing certain 3-generated subuniverses of* $\mathbf{A}^3$ *and checking for the existence of special elements.*


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Cyclic terms

Definition

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Theorem (Lai-Val.)

If $\mathbf{G}$ *is a finite group and* $n > 1$*, then* $\mathbf{G}$ *will have a cyclic term operation of arity* n *if* n *and* $|G|$ *are relatively prime.*

A conjecture

Conjecture

Let Σ be a special Maltsev condition *(i.e., it is strong, idempotent and does not involve compositions of terms). There is a poly-time algorithm to determine, given a finite idempotent algebra $\mathbf{A}$, if $\mathbf{V}(\mathbf{A})$ satisfies Σ.*

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- *It also holds for Gumm terms of a fixed length.*

Related conjectures & questions

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- How hard is it to test if a finite idempotent algebra has a semilattice term operation?

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Questions/Problems

- Is it the case that if Σ is a strong idempotent Maltsev condition that is not equivalent to a special Maltsev condition, then testing for Σ is an EXP-TIME complete problem, even for idempotent algebras?
- How hard is it to test if a finite idempotent algebra has a semilattice term operation?
- Show that if $n > 1$, then there is a poly-time algorithm to test whether a given finite idempotent algebra has an n-ary weak near unanimity term operation.