On Regular Ordered Ternary Semigroups

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Outline

1. Motivation
2. Introduction
3. Main Results
4. References

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3. Main Results
4. References

Motivation

- In 1932 [10], D. H. Lehmer investigated certain ternary algebraic system called triplexes which turn out to be ternary groups.
- In 1965 [15], Zadeh introduced the concept of fuzzy sets. And F. M. Sioson studied ideal theory in ternary semigroups in the same year.
- In 1983 [12], M. L. Santiago developed the theory of ternary semigroups.
- In 1981, 1991 and 1993 [6, 7, 8], fuzzy semigroups have been first considered by Kuroki.

Motivation

- In 2008 [3], T. K. Dutta, S. Kar and B. K. Maity studied some properties of regular ternary semigroups, completely regular ternary semigroups, intra-regular ternary semigroups and characterized them by using various ideals of ternary semigroups.
- In 2012 [5], S. Kar and P. Sarkar introduced and characterized the concept of fuzzy quasi-ideal and fuzzy bi-ideal in ternary semigroups and studied their properties in ternary semigroup.

Outline

1. Motivation
2. Introduction
3. Main Results
4. References

Our aim of this talk is threefold:

1. To characterize regular ordered ternary semigroups in terms of their fuzzy generalized bi-ideals and fuzzy quasi-ideals.
2. To characterize regular ordered ternary semigroups in terms of their fuzzy generalized bi-ideals and fuzzy ideals, fuzzy bi-ideals and fuzzy ideals.
3. To characterize regular ordered ternary semigroups in terms of fuzzy generalized bi-ideals, fuzzy left ideals and fuzzy right ideals and fuzzy bi-ideals, fuzzy left ideals and fuzzy right ideals.

Definition ([13])

A non-empty set S is called a **ternary semigroup** if there exists a ternary operation $[\cdot] : S \times S \times S \to S$ satisfying the following condition:

$$[[abc]de] = [a[bcd]e] = [ab[cde]],$$

for all $a, b, c, d, e \in S$.

For simplicity, we shall write $[abc]$ as abc.

Any semigroup can be reduced to a ternary semigroup. However, S. Banach showed that a ternary semigroup does not necessarily reduce to a semigroup by this examples.

Example

Let $S = \{-i, 0, i\}$. Then S is a ternary semigroup under the multiplication over complex numbers while S is **not** a semigroup under complex number multiplication.

Definition ([1])

A ternary semigroup S is called an **ordered ternary semigroup** if there is a partial order $\leq$ on S such that $a \leq b$ implies $acd \leq bcd$, $cad \leq cbd$ and $cda \leq cdb$, for all $a, b, c, d \in S$.

Example

Let $\mathbb{N}$ be the set of all natural numbers. We define a ternary operation $[\cdot]$ by usual multiplication, that is, $[abc] = abc$ and define a partial order $\leq$ by usual less than or equal to relation. Then $(\mathbb{N}, [\cdot], \leq)$ is an ordered ternary semigroup.

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Definition ([15])

Let X be a non-empty set. A mapping $f : X \to [0,1]$ is called a **fuzzy subset** of X.

Let $a \in S$, we define

$$A_a := \{(x,y,z) \in S \times S \times S : a \le xyz\}$$

and let 1 be a fuzzy subset of S defined as follows

$$1(x) = 1$$

for each $x \in S$.

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$$(\forall a \in S)(f \circ g \circ h)(a) = \begin{cases} \sup\limits_{(u,v,w)\in A_a} \{\min\{f(u), g(v), h(w)\}\} & \text{if } A_a \neq \emptyset, \\ 0 & \text{if } A_a = \emptyset \end{cases}$$

$$f \subseteq g \text{ if and only if } f(x) \leq g(x).$$

$$(f \cap g)(x) = \min\{f(x), g(x)\}$$

and

$$(f \cup g)(x) = \max\{f(x), g(x)\}.$$

Definition ([14])

Let S be an ordered ternary semigroup. A fuzzy subset f of S is called a **fuzzy ternary subsemigroup of** S if for any $x, y, z \in S$,

1. $f(xyz) \geq \min\{f(x), f(y), f(z)\}$ and
2. $x \leq y$ implies $f(x) \geq f(y)$.

Definition ([2])

Let S be an ordered ternary groupoid. A fuzzy subset f of S is called a **fuzzy right [resp. left, lateral] ideal of** S if for any $x, y, z \in S$,

1. $f(xyz) \geq f(x)$ [resp. $f(xyz) \geq f(z)$, $f(xyz) \geq f(y)$] and
2. $x \leq y$ implies $f(x) \geq f(y)$.

We call f a **fuzzy ideal** of an ordered ternary semigroup S if f is a fuzzy right, left and lateral ideal of S.

Definition

Let S be an ordered ternary groupoid. A fuzzy subset f of S is called a **fuzzy quasi-ideal of** S if

1. $(f \circ 1 \circ 1) \cap (1 \circ f \circ 1) \cap (1 \circ 1 \circ f) \subseteq f$,
2. $(f \circ 1 \circ 1) \cap (1 \circ 1 \circ f \circ 1 \circ 1) \cap (1 \circ 1 \circ f) \subseteq f$ and
3. $x \leq y$ implies $f(x) \geq f(y)$ for every $x, y \in S$.

Definition ([14])

Let S be an ordered ternary semigroup. A fuzzy subset f of S is called a **fuzzy generalized bi-ideal of S** if for any $u, v, w, x, y \in S$,

1. $f(uvwxy) \geq \min\{f(u), f(w), f(y)\}$ and
2. $x \leq y$ implies $f(x) \geq f(y)$.

Definition ([14])

Let S be an ordered ternary semigroup. A fuzzy subset f of S is called a **fuzzy bi-ideal of** S if for any $u, v, w, x, y \in S$,

1. f is a fuzzy ternary subsemigroup of S,
2. $f(uvwxy) \geq \min\{f(u), f(w), f(y)\}$ and
3. $x \leq y$ implies $f(x) \geq f(y)$.

An ordered ternary semigroup S is called **regular** [3] if for each $a \in S$ there exists $x \in S$ such that $a \leq axa$.

Outline

1. Motivation
2. Introduction
3. Main Results
4. References

Main Result 1

For an ordered ternary semigroup S, the following assertions are equivalent:
(1) S is regular.
(2) $f = f \circ 1 \circ f \circ 1 \circ f$ for every fuzzy generalized bi-ideal f of S.
(3) $f = f \circ 1 \circ f \circ 1 \circ f$ for every fuzzy bi-ideal f of S.
(4) $f = f \circ 1 \circ f \circ 1 \circ f$ for every fuzzy quasi-ideal f of S.

Main Result 2

Let S be an ordered ternary semigroup. Then the following statements are equivalent:
(1) S is regular.
(2) $f \circ g \circ f \circ g \circ f = f \cap g$ for any fuzzy generalized bi-ideal f and any fuzzy ideal g of S.
(3) $f \circ g \circ f \circ g \circ f = f \cap g$ for any fuzzy bi-ideal f and any fuzzy ideal g of S.

Main Result 3

Let S be an ordered ternary semigroup. Then the following statements are equivalent:
(1) S is regular.
(2) $h \cap f \cap g \subseteq h \circ 1 \circ f \circ 1 \circ g$ for every fuzzy generalized bi-ideal f, every fuzzy left ideal g and every fuzzy right ideal h of S.
(3) $h \cap f \cap g \subseteq h \circ 1 \circ f \circ 1 \circ g$ for every fuzzy bi-ideal f, every fuzzy left ideal g and every fuzzy right ideal h of S.

Outline

1. Motivation
2. Introduction
3. Main Results
4. References

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Thank you for your attention!