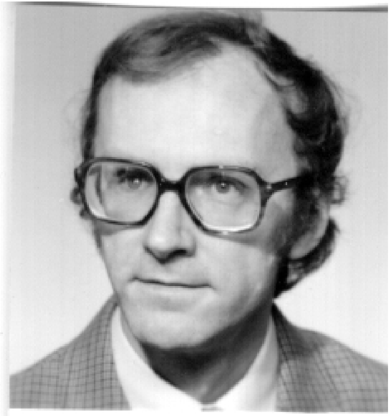
Remembering Jarda



1945–2011

The lattice of equational theories

- J. Ježek: *The lattice of equational theories. Part I: Modular elements.* Czechoslovak Math. J. **31**, 1981, 127–152.
- J. Ježek: *The lattice of equational theories. Part II: The lattice of full sets of terms.* Czechoslovak Math. J. **31**, 1981, 573–603.
- J. Ježek: *The lattice of equational theories. Part III: Definability and automorphisms.* Czechoslovak Math. J. **32**, 1982, 129–164.
- J. Ježek: *The lattice of equational theories. Part IV: Equational theories of finite algebras.* Czechoslovak Math. J. **36**, 1986, 331–341.

Def. An **equational theory** is a set of equations (ordered pairs of terms) of type Δ containing all its consequences (fully invariant congruence).

The lattice $\mathcal{L}_\Delta$ of equational theories is antiisomorphic to the lattice of varieties of Δ-algebras.

Thm. For any type Δ,

- the set of one-based equational theories of type Δ,
- the set of finitely-based equational theories of type Δ,
- the set of equational theories of finite Δ-algebras

is definable in the lattice $\mathcal{L}_\Delta$.

Thm. For any type Δ,

- any finitely based equational theory of type Δ,
- the equational theory of any finite algebra of finite type Δ

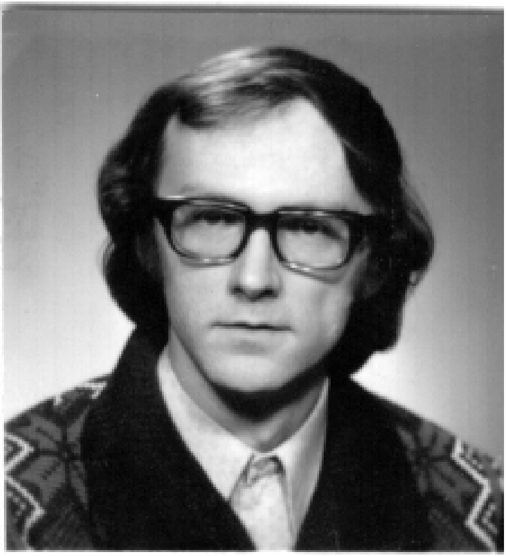
is definable up to automorphisms in $\mathcal{L}_\Delta$.

Thm. If Δ is $\{f\}$ or $\{f, o\}$ for some unary f and nullary o, then the group of automorphisms of $\mathcal{L}_\Delta$ is isomorphic to S_ω, otherwise there are only "syntactically defined" automorphisms.

Further definability results

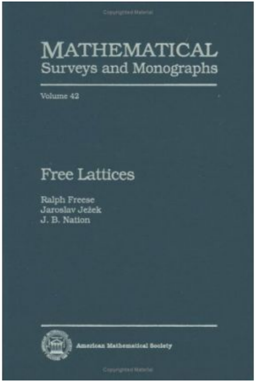
- J. Ježek and G. McNulty: *Bounded and well-placed theories in the lattice of equational theories.* Algebra Universalis **26**, 1989, 311–331.
- J. Ježek and R. McKenzie: *Definability in the lattice of equational theories of semigroups.* Semigroup Forum **46**, 1993, 199–245.
- J. Ježek: *Definability of equational theories of commutative grupoids.* Czechoslovak Math. J. **62**, 2012, 305–333.

- J. Ježek and R. McKenzie: *Definability in substructure orderings, I: finite semilattices.* Algebra Univesalis **61**, 2009, 59-75.
- J. Ježek and R. McKenzie: *Definability in substructure orderings, II: finite ordered sets.* Order **27**, 2010, 115-145.
- J. Ježek and R. McKenzie: *Definability in substructure orderings, III: finite distributive lattices.* Algebra Univesalis **61**, 2009, 283-300.
- J. Ježek and R. McKenzie: *Definability in substructure orderings, IV: finite lattices.* Algebra Univesalis **61**, 2009, 301-312.



Lattice theoretic results

- A. Day and J. Ježek: *The amalgamation property for varieties of lattices.* Transactions of the AMS **286**, 1984, 251–256.
- R. Freese, J. Ježek, J. B. Nation and V. Slavik: *Singular covers in free lattices.* Order **3**, 1986, 39–46.
- R. Freese, J. Ježek and J. B. Nation: *Term rewrite systems for lattices.* J. Symbolic Computation **16**, 1993, 279–288.
- R. Freese, J. Ježek and J. B. Nation: *Lattices with large minimal extensions.* Algebra Universalis **45**, 2001, 221–309.



Minimal big lattices

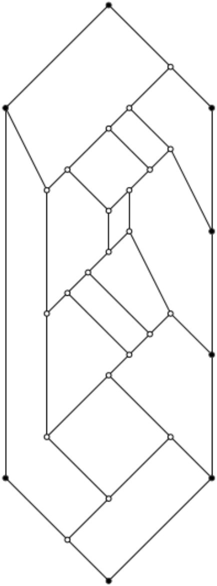
Def. A finite lattice is **big** if it is a maximal sublattice of an infinite lattice, otherwise it is **small**.

Thm. If a finite lattice has a big sublattice, then it is also big.

Thm. There are 145 minimal big lattices. For example, $\mathbf{M}_3$ is big and $\mathbf{N}_5$ is small.

Thm. If $\mathbf{K}$ and $\mathbf{L}$ are small lattices, then their linear sum $\mathbf{K} + \mathbf{L}$ is also small.

Use a general construction to embed $\mathbf{L}$ into a finitely presented lattice generated by a partial lattice.



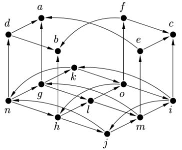
Groupoids and varieties

- J. Ježek and T. Kepka: *The lattice of varieties of commutative abelian distributive groupoids.* Algebra Universalis **5**, 1975, 225–237.
- J. Ježek: *Subdirectly irreducible semilattices with one automorphism.* Semigroup Forum **43**, 1991, 178–186.
- W. Dziobiak, J. Ježek and M. Maróti: *Minimal varieties and quasivarieties of semilattices with one automorphism.* Semigroup Forum **78**, 2009, 253–261.

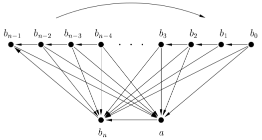
- J. Ježek: *Nonfinitely based three-element idempotent groupoids.* Algebra Universalis **20**, 1985, 292–301.
- J. Ježek, P. Marković, M. Maróti and R. McKenzie: *Equations of tournaments are not finitely based.* Discrete Mathematics **211** (2000), 243–248.
- J. Ježek and R. McKenzie: *The variety generated by equivalence algebras.* Algebra Universalis **45**, 2001, 211–219.
- R. Freese, J. Ježek, P. Jipsen, P. Marković, M. Maróti, R. McKenzie: *The variety generated by order algebras.* Algebra Universalis **47**, 2002, 103–138.

Def. A **tournament** is a conservative commutative groupoid.

$$x \to y \iff xy = yx = x.$$



Thm. The variety generated by tournaments is not finitel based.



Linear representations

Problem: Given $(G, \cdot)$. Is there a (semi-)module M over a (semi-)ring R such that $G \subseteq M$ and

$$x \cdot y = rx + sy + c$$

for some $r, s \in R$, $c \in M$?

Stronger conditions considered, such as $G = M$, $c = 0$, $rs = sr$, r, s have an inverse, etc.

- J. Ježek, *Terms and semiterms.* Commentatationes Math. Univ. Carolinae **20**, 1979, 447–460.
- J. Ježek, T. Kepka: *Linear equational theories and semimodule representations.* International J. of Algebra and Computation **8** (1998), 599–615.

Thm. Every algebra admits a linear representation over a semimodule over a semiring.


15 6:24PM

Linear representations of medial groupoids

A groupoid is medial if $(xy)(uv) = (xu)(yv)$.

Hence, in the representations, we consider *commutative* (semi-)rings.

- J. Ježek, T. Kepka: *Medial groupoids.* Rozpravy ČSAV, Rada mat. a přír. věd **93/2**, 1983, 93 pp.

Thm. Every medial groupoid with $GG = G$ admits a linear representation with $c = 0$ and r, s having an inverse.

Thm. Every cancellative medial groupoid admits a linear representation over a module.

Representations of distributive groupoids

A groupoid is distributive if $x(yz) = (xy)(xz)$ and $(zy)x = (zy)(zx)$.

- J. Ježek, T. Kepka and P. Němec: *Distributive groupoids*. Rozpravy ČSAV, Rada mat. a přír. věd **91/3**, 1981, 94 pp.
- J. Ježek and T. Kepka: *Distributive groupoids and symmetry-by-mediality.* Algebra Universalis **19**, 1984, 208–216.
- J. Ježek and T. Kepka: *Self-distributive groupoids of small orders.* Czechoslovak Math. J. **47**, 1997, 463–468.

Idea. In linear representations, consider "quasi-modules", where $(M, +)$ is a commutative Moufang loop.

Thm. The smallest non-medial distributive groupoid has 81 elements, and there are 6 of them.

Thm. Distributive groupoids are symmetric-by-medial, i.e., there is a decomposition where the blocks are symmetric quasigroups and the factor is medial.

Jarda's work online

www.karlin.mff.cuni.cz/~jezek

- 138 papers
- Universal algebra textbook
- Finite geometries textbook
- Computer programs for calculating with groupoids and lattices