AAA105 - Workshop on General Algebra (105. Arbeitstagung Allgemeine Algebra)

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AAA105 is the 105th edition of "Arbeitstagung Allgemeine Algebra" (Workshop on General Algebra). The conference series was started in 1970 by Rudolf Wille in Darmstadt, Germany. Since then, twice a year researchers meet to exchange their views and results in universal algebra and related topics. The general coordination stayed with Rudolf Wille until 1995, when after 50 conferences he passed it on to Reinhard Pöschel and Bernhard Ganter. From 2013 the series is coordinated by Erhard Aichinger.

AAA105 takes place from May 31st to June 2nd, 2024 at Charles University in Prague and is being organized by Michael Kompatscher and Dmitry Zhuk with the help of Johanka Čablíková, David Stanovský, Libor Barto, Phoebe McDougall, Filippo Spaggiari, Filip Jankovec, Bernardo Rossi, Stefano Fioravanti, Albert Vucaj, Max Hadek and Alexey Barsukov from the Department of Algebra. We also thank the Computer Science Institute of Charles University (IUUK) for their support.

The image on the title page is a drawing by Jaroslav Nešetřil whom we warmly thank for the permission to use it.

Contents

Invited talks

A story of oligomorphic clones and finite minions

Antoine Mottet
TU Hamburg

Universal algebra has found in the recent years an important source of questions and applications in the area of constraint satisfaction problems (CSPs). Much of the so-called "universal-algebraic approach to constraint satisfaction" has been developed within the context of finite (idempotent) algebras, motivated by Feder's and Vardi's dichotomy conjecture, which was eventually proved by Bulatov and Zhuk. Nowadays, two important directions of research in constraint satisfaction concern CSPs with particular infinite templates, and so-called promise CSPs with finite templates. In this talk, I will present the algebraic theory relevant to those fields and show some interesting connections between the two fields.

0-1 laws for finite lattice-ordered algebras

Carles Noguera
University of Siena

In this talk we will concentrate on studying which truth-values are most likely to be taken on finite models by arbitrary sentences of a finitely-valued predicate logic. We show generalizations of Fagin's classical zero-one law for any logic with values in a finite lattice-ordered algebra. We will show a reduction to the classical case through a uniform translation and Oberschelp's generalization of Fagin's result. Moreover, we show that the complexity of determining the almost sure value of a given sentence is PSPACE-complete, and for some logics we may describe completely the set of truth-values that can be taken by sentences almost surely.

This is joint work with Guillermo Badia and Xavier Caicedo.

Equations over finite algebras

Erhard Aichinger
JKU Linz

During the last 20 years, the complexity of solving equations over finite algebras has been studied also from a universal algebraic viewpoint. Solving systems of equations can be seen as a constraint satisfaction problem, which led B. Larose and L. Zádori to a description of algebras in congruence modular varieties for which polynomial systems are solvable in polynomial time. P. Mayr has recently generalized this result to systems of *term equations*. Sytems over supernilpotent algebras can be seen as polynomial systems over finite fields, and we give some new results on the zero sets of such systems (joint work with S. Grünbacher and P. Hametner). The question whether the solutions of one system are contained in the solutions of another system leads to the problem of checking the validity of quasi-identities. We describe the complexity of this problem for algebras with a Mal'cev term (joint work with S. Grünbacher).

Prime Maltsev conditions

Miklós Maróti
University of Szeged

We study the primeness of various Maltsev conditions in the lattice of interpretability types. We will review some known and recent results. In particular, we will show that Taylor varieties and Hobby-McKenzie varieties form prime filters. We will characterize some of these varieties with their graph theoretic properties of their compatible directed graphs. Finally, we will list some open problems. Joint work with B. Bodor, G. Gyenizse, L. Zádori.

The geometry of the word problem for groups and inverse monoids

Robert Gray
University of East Anglia

The most fundamental algorithmic problem in algebra is the word problem, which asks whether there is an algorithm that takes two expressions over a set of generators and decides whether they represent the same element. Despite the huge advances that have been made in this area over the past century, there are still many basic questions about the word problem, and related algorithmic problems, for finitely presented groups and monoids that remain open.

Important work of Ivanov, Margolis, Meakin, and Stephen in the 1990s and 2000s shows how algorithmic problems for groups and monoids can be related to corresponding questions about finitely presented inverse monoids. This is a natural class that lies between groups and monoids, and corresponds to the abstract study of partial symmetries. There is a powerful range of geometric methods for studying inverse monoids such as the Scheiblich/Munn description of free inverse monoids and Stephen's procedure for constructing Schutzenberger graphs.

In this talk I'll explain these connections, and discuss recent advances in our understanding of the behaviour of the geometry of Cayley graphs of inverse monoids, and how this has been used to prove new and unexpected results about their algorithmic and algebraic properties.

Contributed talks

Hölder's theorem for totally ordered monoids

Adam Přenosil
University of Barcelona

Hölder's theorem, one of the early classical results about ordered groups, states that a totally ordered group embeds into the ordered additive group of reals if and only if it is Archimedean. In addition to a fairly routine extension characterizing the totally ordered monoids (tomonoids) which embed into the reals, we show that it also has a more intriguing extension which replaces Archimedeanicity by the property that every congruence is induced by an ideal. More precisely, a congruence of a tomonoid is both a lattice congruence and a monoidal congruence, and an ideal of a tomonoid is both a lattice and a monoidal ideal. Then a tomonoid with no absorbing element has only ideal congruences (that is, every congruence is naturally induced by an ideal) if and only if it embeds either densely or discretely into either the additive group of reals or the additive monoid of non-positive reals. Notice that the division into the dense and the discrete case is implicit already in Hölder's original theorem, since every subgroup of the reals is either dense or discrete.

The quasivariety $\mathbf{SP}(L_6)$. A duality result

Ainur Basheyeva
L.N. Gumilev Eurasian National University
Astana IT University

We prove in [1] that the category of complete bialgebraic $(0,1)$-lattices belonging to the quasivariety $\mathbf{SP}(L_6)$ generated by a finite lattice L_6 with complete $(0,1)$-lattice homomorphisms, is dually equivalent to the category of so-called L_6-spaces with L_6-morphisms. It was established in [2] that the quasivariety $\mathbf{SP}(L_6)$ forms a variety and a finite equational basis for this variety was found. Our proof is based on the approach proposed by W. Dziobiak in [3,4].

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This is a joint work with Marina Schwidefsky.

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The structure of the free F-restriction monoid

Ajda Lemut
University of Ljubljana
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Restriction semigroups are semigroups equipped with two unary operations $^*, ^+$ and are non-regular generalizations of inverse semigroups. For a restriction semigroup S we define a set of projections as $P(S) = \{s^*; s \in S\} = \{s^+; s \in S\}$. We can equip S with a natural partial order $\leq$, where $a \leq b$ holds if and only if there is a projection e such that $a = be$. In such semigroups we have a minimum congruence σ that identifies all projections. Restriction semigroups in which every σ-class has a maximum element are called F-restriction monoids. Thus, F-restriction monoids can be equipped with an additional unary operation $a \mapsto m(a)$. It turns out that F-restriction monoids form a variety in this enriched signature. We show that the free X-generated object $FFR(X)$ in this variety can be decomposed as a semidirect product of an idempotent semilattice of a suitably chosen inverse monoid and the free monoid X^*. This reduces the word problem in $FFR(X)$ to the word problem in a suitable inverse monoid which we show is solvable.

This is joint work with Ganna Kudryavtseva.

Two notions of solvability in Moufang loops

Aleš Drápal
Charles University

To call a loop solvable two iterative approaches may be used. One is based upon the existence of normal abelian subloop (classical solvability). The other approach requires the existence of abelian congruences (congruence solvability). In general a classically solvable loop does not have to be congruence solvable. However, for finite Moufang loops both notions of solvability coincide.

Extensions of R-modules expanded by multilinear operators

Alexander Wires
University of Electronic Science and Technology of China

I report on the deconstruction/reconstruction of extensions in varieties of algebras which are modules expanded by multilinear operators. This is only slightly more general than Kurosh's formalism of Ω-algebras since we consider modules rather than vector spaces. The 1^{st} and 2^{nd}-cohomology is replete with the standard machinery and extensions with abelian ideals agrees with the previous machinery for extensions of affine datum and Schur multiplier results for central extensions in varieties with a difference term. Highlights include a Wells-type description of ideal-preserving automorphisms by a group extension of compatible automorphisms. Derivations (non-cohomological) make sense in these varieties and we similarly see that the ideal-preserving derivations of a group-trivial extension are recovered by a Lie algebra extension. There is a 1-dimensional Hochschild-Serre (or inflation/deflation) exact sequence associated to a general (nonabelian) extension equipped with an additional affine action.

Edge-colourings in graphs and CSPs

Alexey Barsukov
Charles University

My purpose is to introduce the audience to Guarded Monotone Strict NP (GMSNP), a fragment of Existential Second-Order logic (ESO) which strictly extends the Feder and Vardi's logic Monotone Monadic Strict NP (MMSNP). I will discuss how it is related to constraint satisfaction problems (CSPs) and explain the motivation to study the question of "P vs NP-complete" dichotomy with respect to the class GMSNP. Also, I will explain what are the known results for some other questions regarding the logic GMSNP.

The least strict 3-tuple semigroup congruence on the free trioid

Anatoli Zhuchok
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Luhansk Taras Shevchenko National University

Trioids and their linear analogs, trialgebras, first appeared in (J.-L. Loday, M.O. Ronco, Trialgebras and families of polytopes, Contemp. Math. 346, 2004, 369–398). Recall that a *trioid* is a nonempty set T equipped with three binary associative operations $\dashv$, $\vdash$, and $\perp$ satisfying the following axioms:

$$(x \dashv y) \dashv z = x \dashv (y \vdash z), \quad (x \vdash y) \dashv z = x \vdash (y \dashv z),$$

$$(x \dashv y) \vdash z = x \vdash (y \vdash z), \quad (x \dashv y) \dashv z = x \dashv (y \perp z),$$

$$(x \perp y) \dashv z = x \perp (y \dashv z), \quad (x \dashv y) \perp z = x \perp (y \vdash z),$$

$$(x \vdash y) \perp z = x \vdash (y \perp z), \quad (x \perp y) \vdash z = x \vdash (y \vdash z)$$

for all $x, y, z \in T$. A nonempty set G is called a *strict n-tuple semigroup* [1] if, on G, n binary operations are defined, which are denoted by $\boxed{1}, \boxed{2}, \ldots, \boxed{n}$ and satisfy the axioms

$$(x \boxed{r}\, y) \boxed{s}\, z = x \boxed{i}\, (y \boxed{j}\, z) \quad \text{for all} \quad x, y, z \in G \quad \text{and} \quad r, s, i, j \in \{1, 2, \ldots, n\}.$$

If ρ is a congruence on a trioid $(T, \dashv, \vdash, \perp)$ such that $(T, \dashv, \vdash, \perp)/\rho$ is a strict 3-tuple semigroup, we say that ρ is a *strict 3-tuple semigroup congruence*. We characterize the least strict 3-tuple semigroup congruence on the free trioid.

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Decomposition of dyadic polytopes into unions of simplices

Anna Mućka
Warsaw University of Technology

Dyadic n-dimensional convex sets are defined as the intersections with n-dimensional dyadic space of an n-dimensional real convex set. Such a dyadic convex set is said to be a dyadic n-dimensional polytope if the real convex set is a polytope whose vertices lie in the dyadic space. Dyadic convex sets are described as subreducts (subalgebras of reducts) of faithful affine spaces over the ring of dyadic numbers, or equivalently as commutative, entropic and idempotent groupoids under the binary operation of arithmetic mean. Similarly, as in the case of real polytopes, a decomposition (or generalized triangulation) of a dyadic polytope P of dimension n is defined as a decomposition of P into a union of simplices of the same dimension n with pairwise disjoint interiors. We show that, as in the real case, such a decomposition is possible, though the dyadic case is much more complex. We also describe some methods of decomposition.

This is joint work with Anna Romanowska.

Relative annihilators in generalizations of fuzzy structures

Ariane Gabriel Tallee Kakeu
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Non-commutative residuated lattices are the major algebraic counterpart of logics without contraction rule, as they are more generalized logic systems including important classes of algebras behind many-valued and fuzzy logics. Among these, you can find Boolean algebras, pseudo MV-algebras (that is, an algebraic model of the Lukasiewicz infinite valued logic), and pseudo BL-algebras (which are algebras of Hájek's basic (fuzzy) logic), Gödel algebras, pseudo MTL-algebras (algebras of the monoidal t-norm based logic), among others. In the framework of non-commutative residuated lattices, previous works were more focused on filters, whereas the notion of ideal has been introduced in 2017 by Rachunek and Salounova. However the ideal theory in non-commutative residuated lattices have not been deeply explored so far and many interesting questions are still open: the present study seeks to bridge this gap by investigating the missing notion of relative annihilator in non-commutative residuated lattices, along with its properties. Indeed, several studies have been carried out on the basis of (relative) annihilators viewed as ideals in some (commutative) algebraic structures such as rings, distributive lattices, MV-algebras, BL-algebras, De Morgan residuated lattices and residuated lattices. Since ideals and filters are not perfectly dual notions in non-commutative residuated lattices, and considering that the dual of co-annihilator on non-commutative residuated lattices seems not to be an ideal, developing the notion of annihilator that fits the properties sought becomes necessary. This notion has the benefit of generalizing the existing one in subclasses of (non-commutative) residuated lattices. We demonstrate that the set of annihilators of a non-commutative residuated lattice is a complete Boolean algebra, reflecting the results obtained in the particular case of commutative residuated lattices.

This is joint work with Celestin Lele and Lutz Strüngmman.

CSPs for reducts of Johnson graphs

Bertalan Bodor
University of Szeged

The Constraint Satisfaction Problem (CSP) over a structure $\mathfrak{A}$ with a finite relational signature is the problem of deciding whether a given finite structure $\mathfrak{B}$ with the same signature as $\mathfrak{A}$ has a homomorphism to $\mathfrak{A}$. According to the famous result by Bulatov and Zhuk we know that if $\mathfrak{A}$ is finite then the CSP over $\mathfrak{A}$ is either in **P** or it is **NP**-complete. Following this result, it is natural to ask when and how this dichotomy can be generalized for infinite template structures. A currently standing conjecture in this direction is by Bodirsky and Pinsker which states that the same complexity dichotomy holds for first-order reducts of finitely bounded homogeneous structures. A natural subclass of this is the class of all structures which have a first-order interpretation in the pure set. In my talk I present a solution to the dichotomy for all structures in this class which are primitive and have a full interpretation. The case when the template structure is homomorphically equivalent to a structure whose automorphism group is the full symmetric group was handled by Bodirsky and Kára. We show that in all other cases the CSP is **NP**-hard.

This is joint work with Manuel Bodirsky and David Evans.

The Conjugate Elements of a Subsemigroup of I_n

Boonnisa Passararat
University of Potsdam

The set of all injective partial transformations on an n-element set that preserve the following partial order $1 < 2 > 3 < 4 > \ldots n$ forms a semigroup. It is a subsemigroup of the symmetric inverse semigroup which is denoted by IF_n. This partial order is called zig-zag order or fence. A partial transformation α is called fence-preserving if $a < b$ implies $a\alpha \leq b\alpha$. Fence-preserving transformations are studied by several authors since the 90‘s. We will determine all conjugate elements in IF_n. Additional we will provide another algebraic properties of IF_n.

Some Subclasses of Residuated lattices and residuated Multilattices with Applications

Celestin Lele
University of Dschang

A lattice is a poset in which every pair of elements x and y has a l.u.b. $x \vee y$ and a g.l.b. $x \wedge y$. Several generalizations of lattices have been investigated in the literature for their applications, most notably in the area of fuzzy logic programming, coding theory, data management and Formal Concept Analysis. Various special residuated lattices are now used as the main structures of truth values in fuzzy set theory and are subject to intensive algebraic investigation. Some of these generalizations include hyperlattices, nearlattices, multilattices and more recently residuated multilattices Roughly speaking, multilattices are an extension of lattices by allowing multiple suprema and infima subject to a universal property. A residuated multilattice is a partially ordered commutative residuated monoid (a.k.a. pocrim) whose poset is a multilattice. In other words, residuated multilattices combine in a delicate manner the pocrim and multilattice structures on the same set. Therefore residuated multilattices generalize both residuated lattices and multilattices. Cabrera et al. laid the ground work on the topic by introducing the main properties and also studying filters within the new framework. As a generalization of residuated lattices, it seems natural to consider various measures of the gap between the two types of systems. One such measure can be formulated in terms of which additional properties of residuated multilattices would force the structure down to residuated lattices. Note that one instance of such consideration was investigated by Cabrera et al. when they showed that a residuated multilattice with idempotent product is a Heyting algebra. We seek to expand this approach and explore the effect of adding an equation on a residuated multilattice. We shall discover that in some cases, the equation forces the structure to collapse down to the corresponding class of residuated lattice as in the case of the above-mentioned reference. We also obtain in other cases, new classes of residuated multilattices containing examples of pure residuated multilattices and properly containing the corresponding well-known classes of residuated lattices. We hope this work extends and deepens the foundations laid by Cabrera et al. and sets the stage for more in-depth studies of residuated multilattices with applications.

Rings Whose Set of Ideals Form a Semi-Mtl Algebra

Chancelle Olivade Kamga
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It is well known in literature that the set of ideals of a ring forms a residuated lattice. Several authors have characerised rings whose sets of ideals form particular types of residuated lattices, some of them are: Commutative rings whose sets of ideals form MV-algebras investigated by Belluce and non-commutative rings whose sets of ideals form pseudo MV-algebras studied by Di Nola, Kadji, Lele and Nganou. Rings whose sets of ideals form Gödel algebras was studied by Belluce, Di Nola and Marchioni and rings whose sets of ideals form BL-algebras was studied by Heubo-Kwegna, Lele, Ndjeya and Nganou. In this work, we will concentrate on the class of semi-MTL algebras and weak semi-MTL algebras. We will establish some of their properties and study the connection between them and other classes of residuated lattices. By so doing, we will introduce the concepts of semi-prime filters, semi-prime filters of second kind and semi-prime filters of third kind. We will prove that these classes of filters coincide in weak semi-MTL algebras but do not coincide on semi-MTL algebras. Lastly, with these properties and motivated by the review of literature, we will briefly study rings whose sets of ideals form semi-MTL algebras and rings whose sets of ideals form weak semi-MTL algebras.

Nonclassical Polyadic Algebra: Soft and Hard

Chun-Yu Lin
Charles University

This talk characterizes the polyadic algebra arising from algebraic implicative predicate logic, and a functional representation theorem is proved. This connects the soft and hard approaches to algebraizing nonclassical predicate logic.

One paramter derivation on multilattices

Darline Laure Keubeng Yemene
University of Dschang
TU Dresden

A lattice is a poset in which every pair of elements has a least upper bound and a greatest lower bound. Multilattices generalize lattices more clearly, multilattices are structures where the property of being a complete lattice, namely the existence of least upper bound (resp greatest lower bound) for every subset, is weakened to the existence of minimal upper (resp maximal lower) bound. The concept of derivation, which arises from mathematical analysis, was defined for various algebraic structures by using the Leibniz rule. It is useful for studying some structural properties of various kinds of algebra. An algebraic structure with a derivation is broadly called a differential algebra (hence derivation). They are many authors who studied derivations in various algebraic structures. Indeed it has been applied to the theory of commutative rings in 1957 by Posner. For a ring $\mathcal{R} := (R; +, \cdot)$, a map $d : R \to R$ is called a derivation if it satisfies the conditions:

$$\forall x, y \in R \; d(x + y) = d(x) + d(y);$$
$$\forall x, y \in R \; d(x \cdot y) = d(x) \cdot y + x \cdot d(y).$$

It was applied to the theory of lattice by Szász, where the binary ring operations were interpreted as lattice operations $\vee$ and $\wedge$ respectively. Later on, Ferrari investigated some properties of this notion and provided some examples in particular classes of lattices. Xin et al. have defined the notion of derivation on a lattice by considering only the second condition, and they have proven that the first condition holds for isotone derivation on a distributive lattice. In the same paper, they characterized the distributive and modular lattices in terms of their isotone derivation. The notion of derivation has also been generalized in various directions such as Jordan derivation, Lie-derivations, Integral derivation, and Generalized derivation. Motivated by the role played by derivations on lattices in the sense of Xin and coauthors in the application, the aim of the presentation is to introduce the notion of one parameter derivation on multilattices and by calculation, we provide all the one parameter derivations that follow from a given f map and investigate some related properties. We also study some properties of the set of f-fixed points of f-derivations.

Signed Roman Domination in Cartesian Products: Exploring Cycles, Paths, and Complements

Delbrin Ahmed
University of Duhok

A signed Roman dominating function (SRDF) on a graph $G = (V, E)$ is a function $f : V(G) \to \{-1, 1, 2\}$ that satisfies two conditions: (i) the sum of its values over any closed neighborhood is at least one, and (ii) each vertex x where $f(x) = -1$ is adjacent to at least one vertex y where $f(y) = 2$. The weight of an SRDF is the sum of its values over all vertices. The signed Roman domination number of G, denoted $sR(G)$, is the minimum weight of an SRDF on G. In this paper, we investigate the signed Roman domination number of the Cartesian product of graphs, specifically for cycles, paths, and complement graphs.

Strong subalgebras: version 4

Dmitriy Zhuk
Charles University

Many algebraic claims can be proved by induction on the size of the domain of an algebra, but sometimes for induction to work we need to find a subuniverse with stronger properties. For instance, Libor Barto and Marcin Kozik introduced absorbing subuniverses and showed that reduction to absorbing subalgebras preserves linkedness. I introduced four types of strong subalgebras, which was one of the two main ingredients of my proof of the CSP Dichotomy Conjecture. Then, Zarathustra Brady formulated axioms that should be satisfied by a good notion of strong subalgebras and demonstrated their efficiency while studying algebras with bounded width (avoiding linear behavior). I developed a new theory of strong subalgebras that combines most of the good properties of previous theories. It preserves linkedness, captures the linear case but still satisfies most of the axioms formulated by

Zarathustra Brady. This theory finally connected strong subalgebras with bridges, which is the second ingredient of my proof of the CSP Dichotomy Conjecture. In the new theory bridges come naturally from strong subalgebras, which significantly simplified the proof. In the talk we discuss the new theory and demonstrate how it can be used to prove the existence of symmetric term operations.

Effect algebras are nice simplicial sets

Dominik Lachman
Palacký University Olomouc

Effect algebras, in short EA, are traditionally studied as certain partial cancellative monoids. Recently, it was observed that EA are natural examples of Frobenious algebras in the category of relations which we denote FA(**REL**). Another important example of FA(**REL**) are groupoids. EA and groupoids are in some sense two extremal sub-classes of FA(**REL**). By another recent result, we can organise the data defining a Frobenious algebra in **REL** into a simplicial set with some extra structure. While the study of groupoids as simplicial sets is very developed, the case of EA is untouched. In the talk, we consider the case of EA and show that using the combinatorics of simplices, we can capture nicely many concepts essential to EA.

On retract varieties of algebras

Emília Halušková
Slovak Academy of Sciences

A retract variety is defined as a class of algebras closed under isomorphisms, retracts and direct products. D. Jakubíková-Studenovská proved in 1998 that the system of all retract varieties of monounary algebras does not form a set. Each variety of algebras is principal (= generated by one algebra). This is not valid for retract varieties. Dealing with set-principal (= generated by *a set* of algebras) retract varieties, there appear questions as

- Is each retract variety set-principal?
- Is each set-principal retract variety principal?
- Is a system of all set-principal retract varieties a set?

We will deal with monounary algebras and see that the answer to all 3 questions above is no. A description of all set-principal retract varieties of monounary algebras will be given. It is done by using a class $\mathcal{S}$ of some reduced connected monounary algebras. We obtain that

(i) each retract variety generated by a class of connected monounary algebras is generated by a subclass of $\mathcal{S}$.

(ii) each retract variety is generated by algebras that have all connected components from $\mathcal{S}$ and at most two connected components are isomorphic.

This is joint work with Danica Jakubíková-Studenovská. The research is supported by Slovak VEGA grants 1/0152/22 and 2/0104/24.

On the cardinalities of lattices of clonoids

Erkko Lehtonen
Khalifa University

Let C_1 and C_2 be clones on sets A and B, respectively. A set K of functions of several arguments from A to B is called a (C_1, C_2)-*clonoid* if $KC_1 \subseteq K$ and $C_2K \subseteq K$. Sparks classified the clones C on $\{0, 1\}$ according to the cardinality of the lattice $\mathcal{L}_{(\mathsf{J}_A,C)}$ of (J_A, C)-clonoids (here J_A denotes the clone of projections on a finite set A):

- $\mathcal{L}_{(\mathsf{J}_A,C)}$ is finite if and only if C contains a near-unanimity operation;
- $\mathcal{L}_{(\mathsf{J}_A,C)}$ is countably infinite if and only if C contains a Mal'cev operation but no majority operation;
- $\mathcal{L}_{(\mathsf{J}_A,C)}$ has the cardinality of the continuum if and only if C contains neither a near-unanimity operation nor a Mal'cev operation.

We sharpen Sparks's result by finding the cardinality of the lattice of (C_1, C_2)-clonoids for various pairs (C_1, C_2) of clones on the two-element set.

Poset-valued functions and residuated maps

Eszter K. Horváth
University of Szeged

We analyze cuts of poset-valued functions relating them to residuated mappings. Dealing with the lattice-valued case we prove that a function $\mu : X \to L$ induces a residuated map $f : L \to \mathcal{P}(X)$ whose values are the cuts of μ and we describe the corresponding residual. Conversely, it turns out that every residuated map f from L to the power set of X determines a lattice valued function $\mu : X \to L$ whose cuts coincide with the values of f. For general poset-valued functions, we give conditions under which the map sending an element of a poset to the corresponding cut is quasi-residuated, and then conditions under which it is also residuated. We prove that without additional conditions, the map analogue to the residual is a partial function hence we get particular weakly residuated maps which, on the power set of the domain, generate centralized systems instead of closures. We show that the main properties of residuated maps are preserved in this generalized case. We apply these results to the canonical representation of poset-valued and lattice-valued functions, using the corresponding closures and centralized systems.

Joint work with Sándor Radeleczki, Branimir Šešelja, and Andreja Tepavčević.

Subvariety lattice of the variety of pointed Abelian l-groups

Filip Jankovec
Charles University

The variety of lattice-ordered Abelian groups (Abelian l-groups, for short) is well known and studied. It was established that, as a quasivariety, Abelian l-groups are generated by the l-group of integer numbers and thus it does not contain any proper subquasivarieties. Abelian l-groups are not only interesting from the algebraic point of view but also from the logical point of view, since they form the algebraic semantics for Abelian logic.

Although the lattice of sub(quasi)varieties of the variety of Abelian l-groups is trivial, it turns out that simply considering pointed structures yields a much more interesting structure. It has been shown that the lattice of subvarieties of MV-algebras embeds into the lattice of subvarieties of pointed Abelian l-groups via a generalized version of the Mundici functor. Recall that the lattice of subvarieties of MV-algebras was described by Komori in 1981. The aim of this talk is to present a description of the lattice of subvarieties of pointed Abelian l-groups together with an equational presentation.

Disorder in Algebra

Filippo Spaggiari
Charles University

How predictable is the multiplication table of a given binary algebraic structure? When the algebra is defined by specific identities, these equations impose restrictions on the table. Yet, to what extent can we challenge these constraints? This presentation delves into the spectrum of disorder in given classes of algebraic structures, such as one-sided quasigroups and quandles, spanning from trivial cases to Latin squares. Additionally, it explores how this disorder behaves in the context of universal algebraic constructions such as subalgebras, products, and homomorphic images. The key analytical tool for this exploration is the recently developed concept of the entropy function, together with its distinctive properties.

Weakest non-trivial finite Structures

Florian Starke
TU Dresden

It is well known that the pp-constructability poset has a unique coatom. It can for example be represented by the digraph $\mathbb{P}_1$ consisting of a single directed edge. In this talk I will characterise the class of finite structures that are pp-equivalent to $\mathbb{P}_1$ with a set of h1-identities, a Datalog fragment, and a description of the obstructions.

A new approach to universal F-inverse monoids in enriched signature

Ganna Kudryavtseva
University of Ljubljana
IMFM Ljubljana

We show that the universal X-generated F-inverse monoid $F(G)$, where G is an X-generated group, introduced by Auinger, Szendrei and the first-named author, arises as a quotient inverse monoid of the Margolis-Meakin expansion $M(G, X \cup \overline{G})$ of G, with respect to the extended generating set $X \cup \overline{G}$, where $\overline{G}$ is a bijective copy of G which encodes the m-operation in $F(G)$. This involves some closure operator on the semilattice of all finite and connected subgraphs containing the origin of the Cayley graph $\mathrm{Cay}(G, X \cup \overline{G})$ and leads to a new and simpler proof of the universal property of $F(G)$.

This is joint work with Ajda Lemut Furlani.

Decision problems from idempotent semirings to substructural logics

Gavin St. John
University of Salerno

The decidability of a given logic reveals the expressive power and tractability of its theory. A logic is decidable if there is an algorithm that can determine whether an input formula is a theorem of the logic or not, known as the *provability problem*. More generally, there is the *deducibility problem*: is an input formula a logical consequence of an input set of assumptions? If such a procedure exists, in either case, its computational complexity refers to the amount of time and space required to run the decision algorithm as a function of the input size. This work contributes to this line of research providing complexity lower bounds, or *hardness* results, for a vast collection of substructural logics through their extensions by simple structural rules. Simple structural rules correspond, algebraically, to equations in the signature of the join and multiplication connectives in residuated lattices - the fragment of which forms an idempotent semiring. The reductions involve an encoding of computations of counter machines in special idempotent semirings that are resilient to the effect of certain simple equations. Through a construction involving residuated frames, a residuated lattice is produced, satisfying the relative axiomatizations, which is used to establish the completeness of encoding the decision problems.

This is joint work with Nick Galatos, Vitor Greati, and Revantha Ramanayake.

A restricted class of majority functions and minimal clones

Hajime Machida

The target of this talk is majority functions f which are 2-valued on the set of triples with mutually distinct components. For brevity, we shall call them 2-*valued*. We define the concepts of orderly and doubly orderly for them; f is *orderly* if it satisfies $f(x, y, z) \approx f(y, z, x) \approx f(z, x, y)$ and it is *doubly orderly* if, moreover, it satisfies $f(x, y, z) \approx f(z, y, x)$ (i.e., totally symmetric). We prove the following. For a 2-valued majority function f, the clone generated by f always contains an orderly majority function. With additional assumptions imposed on some values of f, the clone generated by f contains a doubly orderly majority function. Based on these, and some more detailed, observations, we establish a class of minimal functions.

A characterization of permutability of 2-uniform tolerances on posets

Helmut Länger
TU Wien
Palacký University Olomouc

Tolerance relations were investigated by several authors in various algebraic structures, see e.g. the monograph by I. Chajda. Recently G. Czédli studied so-called 2-uniform tolerances on lattices, i.e. tolerances that are compatible with the lattice operations and whose blocks are of cardinality 2. He showed that two such tolerances on a lattice containing no infinite chain permute if and only if they are amicable (a concept introduced in his paper). We extend this study to tolerances on posets. Since in posets we have no lattice operations, we must modify the notion of amicability. We modified it in such a way that in case of lattices it coincides with the original definition. With this new definition we can prove that two 2-uniform tolerances on a poset containing no infinite chain permute if and only if they are amicable in the new sense.

Orthogonality and complementation in the lattice of subspaces of a finite vector space

Ivan Chajda
Palacký University Olomouc

We investigate the lattice $L(V)$ of subspaces of an m-dimensional vector space V over a finite field $GF(q)$ together with the unary operation of orthogonality. It is known that this lattice is modular and that orthogonality is an antitone involution.The lattice $L(V)$ satisfies the chain condition and we determine the number of covers of its elements, in particular the number of its atoms. We characterize when orthogonality is a complementation and hence $L(V)$ is orthomodular.

Short definitions in constraint languages

Jakub Bulín
Charles University

A first-order formula is called *primitive positive (pp)* if it only admits the use of existential quantifiers and conjunction. Pp-formulas are a central concept in (fixed-template) constraint satisfaction since CSP(Γ) can be viewed as the problem of deciding the primitive positive theory of Γ, and pp-definability captures gadget reductions between CSPs. An important class of tractable constraint languages Γ is characterized by having *few subpowers*, that is, the number of n-ary relations pp-definable from Γ is bounded by $2^{p(n)}$ for some polynomial $p(n)$. In this talk we discuss a restriction of this property, stating that every pp-definable relation is definable by a pp-formula of polynomial length. We conjecture that the existence of such *short definitions* is actually equivalent to Γ having few subpowers, and verify this conjecture for a large subclass that, in particular, includes all constraint languages on three-element domains. We furthermore discuss how our conjecture imposes an upper complexity bound of co-NP on the subpower membership problem of algebras with few subpowers.

This is joint work with Michael Kompatscher.

Linear quasigroups and quasigroup triality.

Jonathan D. H. Smith
Iowa State University

Linearization is a standard trick in universal algebra. A universal algebra (A, F) is linear if the underlying set A is a module over a commutative, unital ring S, and each basic operation $f(x_1, \ldots, x_n)$ is a linear combination $x_1X_1 + \ldots + x_nX_n$ of the variables $x_1, \ldots, x_n$ acted upon by successive S-endomorphisms of A. The identities of algebras in a variety lead to relations on the various S-endomorphisms. Linear quasigroups are equivalent to representations of two-generated groups. However, the usual expression of this equivalence does not fit well with the full S_3 or triality symmetry of the language of quasigroups. We present a new homogeneous algebra which does provide a representation of linear quasigroups that is naturally invariant under the triality symmetry. The quaternion algebra over S appears as a quotient of the homogeneous algebra. Modules over this quaternion algebra are equivalent to linear quasigroups satisfying a set of quasigroup identities, which are permuted under triality.

End-regular flower graphs $C_{r,s}$

Jörg Koppitz
Bulgarian Academy of Sciences

We will present the regularity of the endomorphism monoid of a class of graphs, which generalizes the cycle graph. We give a complete list of all end-regular $r \times s$ flower graphs. In particular, for an odd integer $s \geq 3$, the $s \times s$ flower graph $C_{s,s}$ has a regular endomorphism monoid, i.e., $C_{s,s}$ is end-regular. For $C_{s,s}$, $s \geq 3$ is odd, we determine the cardinality and the rank (including a generating set of minimal size).

Measuring associativity of graph algebras

Kamilla Kátai-Urbán
University of Szeged

We study two measures of associativity for graph algebras of finite undirected graphs: the index of nonassociativity and (a variant of) the semigroup distance. We determine "almost associative" and "antiassociative" graphs with respect to both measures. We compare the results with a third measure of associatvity, the associative spectrum, which was previously investigated for graph algebras by Tamás Waldhauser and Erkko Lehtonen.

This is joint work with Tamás Waldhauser.

One-dimensional strong affine representations of the polycyclic monoids

Kristóf Varga
University of Szeged

The *polycyclic monoid* $\mathcal{P}_n$ is a monoid with zero given by the presentation

$$\mathcal{P}_n = \langle a_0, \ldots, a_{n-1}, a_0^{-1}, \ldots, a_{n-1}^{-1} : a_i^{-1} a_i = 1 \text{ and } a_i^{-1} a_j = 0, i \neq j \rangle.$$

Let $D = (d_0, d_1, \ldots, d_{n-1})$ be a complete system of residues modulo n and let us consider the functions $f_i \colon \mathbb{Z} \to \mathbb{Z}, x \mapsto nx + d_i$ $(i = 0, 1, \ldots, n-1)$. These functions give rise to a so-called one-dimensional strong affine representation of the polycyclic monoid $\mathcal{P}_n$. This representation can be visualized by an edge-labeled directed graph: we draw an arrow with label i from x to $f_i(x)$ for each $x \in \mathbb{Z}$ and $i \in \{0, 1, \ldots, n-1\}$.

Every connected component of such a graph contains exactly one cycle that can be uniquely identified by the word obtained by recording the labels along the edges of the cycle. It can be shown that the set of words corresponding to the cycles determine the edge-labeled graph up to isomorphism as well as the representation up to equivalence. Our main problem is describing the set of words corresponding to a one-dimensional strong affine representation of $\mathcal{P}_n$. It has been proven that every finite set of words can be extended to obtain a set of words describing a representation, and we have previously shown how to do it if the system of residues is an arithmetic sequence starting with zero. This talk will focus on a new result, where we have found a complete characterization for the sets of words induced by arbitrary arithmetic sequences (i.e., if $D = (c, c+h, c+2h, \ldots, c+(n-1)h)$ for some positive integers c and h where h is relatively prime to n), and that these sets (together with the empty set) form a closure system in the set of words over $\{0, 1, \ldots, n-1\}$. Furthermore, we will provide a method for obtaining the closure of an arbitrary set of words.

Some properties of reduced and semiprime rings

Małgorzata Jastrzębska
Military University of Technology

Let R be an associative ring. An element a in R is said to be nilpotent if there exist positive integer n such that $a^n = 0$. A ring is called a reduced ring if it has no non-zero nilpotent elements. In the commutative case, this is equivalent to R being a semiprime ring. If X is a subset of R then $l(X)$, set of all a from R, such that $aX = 0$ is said to be the left annihilator of X in R. Then the set of all left annihilators in R is a subset of the set of all left ideals in R. This set is a lattice but need not be a sublattice of the lattice of left ideals. Analogous fact is true for right annihilators and right ideals in R. In this talk we are going to present some results about reduced rings related to lattices of annihilators.

Some Sufficient Conditions for Bounded Width over Infinite Structures

Michał Wrona
Jagiellonian University

We consider natural classes of structures $\mathbb{A}$ with finite duality or whose age has the free amalgamation property and their first order expansions $\mathbb{B}$. Provided $\mathbb{B}$ is preserved by a chain of quasi Jónsson operations, we show that it has bounded width.

Towards decomposing lattices of integer partitions using arrow relations

Mike Behrisch
TU Wien

Integer partitions represent ways to write a positive integer as the sum of smaller positive integers, e.g., $(2, 2, 1)$ is a partition of $n = 2+2+1 = 5$. Brylawski showed that for a fixed $n \in \mathbb{N}$ these partitions can be ordered by dominance, under which they become a complete lattice $\mathcal{L}_n$. A classic result of Hardy and Ramanujan shows that the sequence of sizes $(|\mathcal{L}_n|)_{n\in\mathbb{N}}$, which is `A000041` in Sloane's On-line Encyclopedia of Integer Sequences, grows asymptotically as $\exp(\pi\sqrt{2n/3})/(4\sqrt{3}n)$. The increasingly more complicated lattices $\mathcal{L}_n$ can be described more compactly by their so-called standard contexts $(\mathcal{J}(\mathcal{L}_n), \mathcal{M}(\mathcal{L}_n), \leq)$, where $\mathcal{J}(\mathcal{L}_n)$ denotes the set of completely join-irreducible and $\mathcal{M}(\mathcal{L}_n)$ the set of completely meet-irreducible elements of $\mathcal{L}_n$. In the talk we discuss how so-called arrow relations between $\mathcal{J}(\mathcal{L}_n)$ and $\mathcal{M}(\mathcal{L}_n)$ can be used to obtain subdirectly irreducible factors for subdirect representations of $\mathcal{L}_n$, and we present descriptions of these arrow relations for all $n \in \mathbb{N}$.

This is joint work with Asma Almazaydeh, Edith Vargas García, and Andreas Wachtel.

Congruence-simple matrix semirings

Miroslav Korbelář
Czech Technical University in Prague

It is well known that the full matrix ring over a skew-field is a simple ring. We generalize this theorem to the case of semirings. We characterize the case when the matrix semiring $M_n(S)$, of all $n\times n$ matrices over a semiring S, is congruence-simple, provided that either S is commutative or S is finite or S is downwards directed (in the sense of the natural semiring quasiorder). We also investigate in this context the case when S is additively idempotent and $xy + yx + x = x$ for all $x, y \in S$.

This is joint work with V. Kala and T. Kepka.

Natural representation of lattices and other algebras

Miroslav Ploščica
Šafárik University Košice

In 1994, the author has developed a representation theory for lattices, in the spirit of the theory of natural dualities. The representation is based on the concept of maximal partial homomorphisms into the generating object. In the present talk we discuss the possibility to generalize this approach and apply it to other kinds of algebras. We identify problems and difficulties and suggest ways to overcome them.

Calculating extended commutator equalities

Nebojša Mudrinski
University of Novi Sad

Once the commutator of arbitrary finite length on the congruence lattice of an algebra has been introduced by Bulatov 2001, a natural question has come up: Can the commutators of all length be given by finitely many equations? More precisely, can one recover all the values of all the commutators if we fix the value of finitely many of those commutators for finitely many congruences? Unfortunately, it turned out that if we fix finitely many values of operations in an operation sequence that satisfies the omission property (HC3) and symmetry (HC4) of higher commutators then such an operation sequence is not uniquely determined. Therefore, extended commutators have been introduced in an earlier paper. There it has been proved that an operation sequence that satisfies (HC3) and (HC4) is uniquely determined by finitely many extended commutator equalities. Unfortunately to calculate the value of extended commutator one need to calculate infinitely many values of higher commutators. This problem can be reduced to calculating just finitely many values of higher commutators under certain additional conditions.

The growth of free inverse monoids

Nóra Szakács
University of Manchester

We compute the rate of exponential growth of the free inverse monoid of rank r (and hence an upper bound on the corresponding rate for all r-generated inverse monoids and semigroups). This turns out to be an algebraic number strictly between the obvious bounds of $2r - 1$ and $2r$, tending to $2r$ as the rank tends to infinity. We also find an explicit expression for the exponential growth rate of the number of idempotents, and prove that this tends to $\sqrt{e(2k-1)}$ as $k \to \infty$.

Joint work with Mark Kambites, Carl-Fredrik Nyberg-Brodda, and Richard Webb.

Minimal operations over permutation groups

Paolo Marimon
TU Wien

We classify the operations which are minimal above the clone generated by a permutation group G acting on a set. Rosenberg (1986) showed that when G is trivial there are only five types of minimal operations. We show that for most permutation groups there are only three types of minimal operations. More generally, we carry such classification for any permutation group and specify the behaviour of the minimal operations on tuples with two or more elements in the same orbit. We improve and generalise results of Bodirsky and Chen (2007) for oligomorphic permutation groups. Moreover, our classification allows us to obtain positive results helpful to the study of constraint satisfaction problems on omega-categorial structures.

This is joint work with Michael Pinsker.

Bulatov's colored edge theory in minimal Taylor algebras

Petar Marković
University of Novi Sad

We limit Bulatov's colored edge theory which led to his proof of the Dichotomy Conjecture for the CSP from "smooth algebras", where he originally defined it, to minimal Taylor algebras. In this narrower setting we are able to obtain the following results:

(i) A much easier and more intuitive definition of directed edges of the graphs in a pseudovariety generated by a minimal Taylor algebra.

(ii) A set of important properties, which we call "Edge Axioms", satisfied both by Bulatov's and our edges, which are sufficient for all subsequent proofs, and which allow us to play with the definition of edges in the future.

(iii) A simplification of some proofs in Bulatov's theory using the established facts about minimal Taylor algebras, and also a simplification of some proofs about minimal Taylor algebras using the previously reproved Bulatov's results.

We finish the lecture with some open problems.

The presented results were obtained in collaboration with Z. Brady, P. Djapic, M. Kozik, A. Prokic and V. Uljarevic

Automorphism groups of filtered Boolean powers

Peter Mayr
University of Colorado Boulder

Let A be a finite simple non-abelian Mal'cev algebra (e.g. a group, loop, ring). We characterize the Fraïssé limit of the class of finite direct powers of A as a filtered Boolean power of A by the countable atomless Boolean algebra B. In general, such a filtered Boolean power of A by B is ω-categorical and its automorphism group G is a Polish group under pointwise convergence. We show that

- G has the small index property, i.e., every subgroup of G of index less than $2^{\aleph_0}$ is open;
- G has uncountable cofinality, i.e., it is not the union of a countable chain of proper subgroups;
- G has the Bergman property, i.e., for each generating set E of G with $1 \in E = E^{-1}$ there exists $k \in \mathbb{N}$ such that $E^k = G$.

This is joint work with Nik Ruškuc.

Skew braces - a new playground for algebraists

Péter P. Pálfy
Rényi Institute

Skew braces were introduced by Guarnieri and Vendramin in 2017, generalizing an earlier concept by Rump. They are related to the study of the Yang–Baxter equation in theoretical physics. A skew brace is an algebra $(A; +, \circ)$, where both $(A; +)$ and $(A; \circ)$ are groups (despite what the notation suggests, the operation $+$ need not be commutative), and the identity $x \circ (y + z) = x \circ y - x + x \circ z$ is satisfied. While there are lots of papers on skew braces written by group theorists, this new algebraic structure has not arouse the interest but a few universal algebraists. I am going to discuss some open problems concerning skew braces.

Generalized quasiorders, in particular 1- and 2-dimensional on a 2-element set

Reinhard Pöschel
TU Dresden

A 1-dimensional generalized quasiorder $\varrho \subseteq A^m$ has the property that an operation $f : A^n \to A$ preserves ϱ if (and only if) each translation of f preserves ϱ (a translation is a unary function obtained from f by substituting constants). We introduce d-dimensional generalized quasiorders satisfying the analogous property with d-translations (i.e., d-ary functions obtained from f by substituting constants) instead of translations. In this talk we focus on 1- and 2-dimensional generalized quasiorders on a 2-element set. For this case we characterize and describe all 1-dimensional and many 2-dimensional generalized quasiorders

This is joint work with Danica Jakubíková-Studenovská and Sándor Radeleczki, and Andrew Moorhead, respectively.

Fresh insights into power graphs of infinite groups

Samir Zahirović
University of Novi Sad

The *directed power graph* $\vec{\mathcal{P}}(\mathbf{G})$, the *power graph* $\mathcal{P}(\mathbf{G})$, and the *enhanced power graph* $\mathcal{P}_e(\mathbf{G})$ of a group are graphs with vertex set G such that $x \to y$ in $\vec{\mathcal{P}}(\mathbf{G})$ if $y \in \langle x \rangle$, $x \sim y$ in $\mathcal{P}(\mathbf{G})$ if $y \in \langle x \rangle$ or $x \in \langle y \rangle$, and $x \sim y$ in $\mathcal{P}_e(\mathbf{G})$ of $\langle x, y \rangle$ is cyclic. The power graph, the directed power graph, and the enhanced power graph of every finite group determine each other. Cameron, Guerra and Jurina later proved that, if two torsion-free groups $\mathbf{G}$ and $\mathbf{H}$ of nilpotency class 2 have isomorphic power graphs, then $\vec{\mathcal{G}}(\mathbf{G}) \cong \vec{\mathcal{G}}(\mathbf{H})$. In this talk, we delve into more recent findings regarding the interconnection of these graphs within the context of infinite groups.

Passive structural completeness in quasivarieties of logic

Sara Ugolini
IIIA - CSIC

A quasivariety Q is structurally complete if every quasiequation that is admissible (i.e., such that any substitution that makes its premises valid, also makes the conclusion valid) is valid in Q. The notion of structural completeness comes from logic: a logic is structurally complete if each of its proper extensions admits new theorems, or equivalently, if and only if all of its admissible rules are derivable. It is well-known that classical logic is structurally complete and intuitionistic logic is not. However, intuitionistic logic satisfies a weaker notion, that is, it is passively structurally complete: all of its passive admissible rules (i.e. those for which there is no substitution making their premises a theorem) are derivable. In this contribution we present some new algebraic characterizations of passive structural completeness for quasivarieties in general, and we then apply them to (quasi)varieties of bounded integral residuated lattices (which are the equivalent algebraic semantics of substructural logics with weakening). Seen from the logical point of view, we show that a substructural logic satisfying the weakening rule is passively structurally complete if and only if every contradiction of classical logic is explosive in it.

Polymorphisms in CSPs, topology and social choices

Sebastian Meyer
TU Dresden

The polymorphisms of a structure famously classify the complexity class of the corresponding constraint satisfaction problem. Polymorphisms are also known to give invariants of topological spaces up to homotopy. In a different area of mathematics, polymorphisms are called social choice functions and are studied with completely different backgrounds. I consider the question which polymorphisms make a topological space contractible and present (often overlapping) results and applications in multiple directions.

Testing sparse polynomial identities over finite fields

Simon Grünbacher
JKU Linz

We want to determine whether a multivariate polynomial $p \in \mathbb{F}[X_1, \ldots, X_n]$ vanishes on a grid $S^n \subseteq (\mathbb{F} \setminus \{0\})^n$. We give conditions under which this question can be answered by evaluating p on a small *testing set* $T \subseteq S^n$. We prove the following result:

> Let $\mathbb{F}$ be a field with $q > 2$ elements, let $t := \frac{q-1}{q-2}$, let $S \subseteq \mathbb{F} \setminus \{0\}$ and let $m \in \mathbb{N}$. Then there exists a testing set $T \subseteq S^n$ of size at most $(n \cdot$

$|S|)^{\log_t(m)}$ such that for all $p \in \mathbb{F}[X_1, \ldots, X_n]$ with $M(p) \leq m$ monomials, we have

$$(\forall x \in S^n : p(x) = 0) \iff (\forall x \in T : p(x) = 0).$$

This is joint research with Erhard Aichinger and Paul Hametner.

Möbius Inversion on Action Networks

Stefan E. Schmidt
TU Dresden

Looking up Möbius Inversion on German Wikipedia gives a mainly number-theoretic view. The English version extends Möbius Inversion to locally finite partially ordered sets as a fundamental combinatorial tool of inclusion-exclusion. Under the umbrella of so-called Action Networks (defined as small covariant categories) both approaches will be shown to be comparable within a geometric setting. The main mathematical structure which gives clarification is that of the Full Convolution Algebra of a naturally ordered split-finite action network.

Varieties of MV-monoids and positive MV-algebras

Stefano Fioravanti
Charles University

We investigate MV-monoids and their subquasivarieties. MV-monoids are algebras $\langle A, \vee, \wedge, \oplus, \odot, 0, 1\rangle$ where $\langle A, \vee, \wedge, 0, 1\rangle$ is a bounded distributive lattice, $\langle A, \oplus, 0\rangle$ and $\langle A, \odot, 1\rangle$ are commutative monoids, and some further connecting axioms are satisfied. Every MV-algebra in the signature $\{\oplus, \neg, 0\}$ is term equivalent to an algebra that has an MV-monoid as a reduct, by defining, as standard, $1 := \neg 0$, $x \odot y := \neg(\neg x \oplus \neg y)$, $x \vee y := (x \odot \neg y) \oplus y$ and $x \wedge y := \neg(\neg x \vee \neg y)$. Particular examples of MV-monoids are positive MV-algebras, i.e. the $\{\vee, \wedge, \oplus, \odot, 0, 1\}$-subreducts of MV-algebras. Positive MV-algebras form a peculiar quasivariety in the sense that, albeit having a logical motivation (being the quasivariety of subreducts of MV-algebras), it is not the equivalent quasivariety semantics of any logic. We study the lattice of subvarieties of MV-monoids and describe the lattice of subvarieties of positive MV-algebras. We characterize the finite subdirectly irreducible positive MV-algebras. Furthermore, we axiomatize all varieties of positive MV-algebras.

Joint work with Marco Abbadini and Paolo Aglianò.

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Monoidal intervals on finite sets

Tamás Ágoston

University of Szeged

Let A be a finite set, and M be a transformation monoid on A. Then the collection of all clones on A whose set of unary operations coincides with M form an interval in the lattice of all clones on A. There are still open problems about these intervals, even in the 3-element case. We are mainly working on three-element sets, and we will present new families of finite monoidal intervals.

Associativity conditions for linear quasigroups

Tamás Waldhauser

University of Szeged

We study associativity conditions of the form $t_1(x_1, \ldots, x_n) \approx t_2(x_1, \ldots, x_n)$, where t_1 and t_2 are linear groupoid terms such that the variables $x_1, \ldots, x_n$ appear in this order in t_1 and t_2. Thus t_1 and t_2 are obtained from $x_1 \circ \cdots \circ x_n$ by inserting round brackets; hence we call these identities *bracketing identities*. The set of all bracketing identities satisfied by a groupoid is called the *fine associative spectrum*, as it accounts for the consequences of the associative law that hold in the given groupoid. Belousov proved in 1966 that a quasigroup $\mathbf{A} = (A, \circ)$ satisfying a nontrivial bracketing identity must be an *affine quasigroup* over some group, i.e., $x \circ y = \varphi(x) + c + \psi(y)$, where $(A, +)$ is a (not necessarily Abelian) group, φ, ψ are automorphisms of this group and $c \in A$ is an arbitrary constant. Here we focus on the special case $c = 0$, i.e., on *linear quasigroups* over groups. We characterize the bracketing identities satisfied by such linear quasigroups with the help of certain equivalence relations on binary trees that are based on the left and right depths of the leaves modulo some integers. We prove that the set of all fine associative spectra of linear quasigroups forms a lattice, and we describe the structure of this lattice. In particular, we determine the atoms and the coatoms, thereby characterizing the “least associative” and the “most associative” (yet not associative) linear quasigroups.

This is a joint work with Erkko Lehtonen.

On three submonoids of the dihedral inverse monoid on a finite set

Teresa Melo Quinteiro
NOVA Math
ISEL

Let Ω_n be a finite set with n elements ($n \in \mathbb{N}$), say $\Omega_n = \{1, 2, \ldots, n\}$. Denote by Sym_n the *symmetric group* on Ω_n, i.e. the group (under composition of mappings) of all permutations on Ω_n, and by I_n the *symmetric inverse monoid* on Ω_n, i.e. the inverse monoid (under composition of partial mappings) of all partial permutations on Ω_n. From now on, suppose that Ω_n is a chain, e.g. $\Omega_n = \{1 < 2 < \cdots < n\}$. An element $\alpha \in \mathrm{I}_n$ is called *order-preserving* [*order-reversing*] if $x \leqslant y$ implies $x\alpha \leqslant y\alpha$ [$x\alpha \geqslant y\alpha$], for all $x, y \in \mathrm{dom}(\alpha)$. A partial permutation is said to be *monotone* if it is order-preserving or order-reversing. Let $s = (a_1, a_2, \ldots, a_t)$ be a sequence of t ($t \geqslant 0$) elements from the chain Ω_n. We say that s is *cyclic* [*anti-cyclic*] if there exists no more than one index $i \in \{1, \ldots, t\}$ such that $a_i > a_{i+1}$ [$a_i < a_{i+1}$], where a_{t+1} denotes a_1. We also say that s is *oriented* if s is cyclic or s is anti-cyclic. Given a partial permutation $\alpha \in \mathrm{I}_n$ such that $\mathrm{dom}(\alpha) = \{a_1 < \cdots < a_t\}$, with $t \geqslant 0$, we say that α is *orientation-preserving* [*orientation-reversing*, *oriented*] if the sequence of its images $(a_1\alpha, \ldots, a_t\alpha)$ is cyclic [anti-cyclic, oriented]. For a long time, monoids of order-preserving partial permutations and orientation-preserving partial permutations have aroused interest in various authors, who studied Green's relations, cardinalities, ideals, congruences, generators and ranks, maximal subsemigroups, presentations, automorphisms and endomorphisms, etc. For $n \geqslant 3$, consider the well-known *dihedral group* D_{2n} of order $2n$:

$$\begin{aligned}\mathrm{D}_{2n} &= \langle g, h \mid g^n = 1, h^2 = 1, hg = g^{n-1}h \rangle \\ &= \{\mathrm{id}, g, g^2, \ldots, g^{n-1}, h, hg, hg^2, \ldots, hg^{n-1}\},\end{aligned}$$

where $g = (1\ 2\ \cdots\ n)$, h is given by $h\colon i \mapsto n - i + 1$ and id denotes the identity mapping on Ω_n. The elements of I_n are precisely all restrictions of permutations on Ω_n. For $n \geqslant 3$, if we consider only restrictions of permutations on Ω_n that belong to the dihedral group D_{2n}, we obtain the inverse submonoid DI_n of I_n, named by Fernandes and Paulista by *dihedral inverse monoid on* Ω_n. Notice that, it is clear that, for any subgroup G of Sym_n, the set $\mathrm{I}_n(G)$ of all restrictions of elements of G forms an inverse submonoid of I_n whose group of units is precisely G. Fernandes and Paulista studied the monoid DI_n by determining its cardinality and rank as well as descriptions of its Green's relations and presentations for DI_n. In this talk we consider the submonoids of the dihedral inverse monoid DI_n, that arise when we consider all orientation-preserving, monotone or order-preserving elements of DI_n. For each of these three monoids, we compute the cardinal, give descriptions of Green's relations, determine the rank and exhibit presentations.

This is a joint work with I. Dimitrova, V.H. Fernandes and J. Koppitz.

The meet-stalactic monoid

Thomas Aird
University of Manchester

The left and right stalactic monoids are defined by equating words which produce the same tableau under some insertion algorithm. In this talk, I introduce the meet-stalactic monoid, which is defined by equating words which are equal in both the left and right monoids. We discuss several interesting properties of this monoid, including a Robinson-Schensted-like result.

Permutation clones that preserve relations

Tim Boykett
Time's Up
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Permutation clones generalise clones to permutations of A^n. Emil Jeřábek found the dual structure to be weight mappings $A^k \to M$ to a commutative monoid, generalising relations. We investigate the case when the dual object is precisely a relation, equivalently, that $M = \mathbb{B}$, calling these relationally defined permutation clones. We use connections between clones and permutation groups. We determine the number of relationally defined permutation clones on two elements (13) and find that many classes of infinite collections of clones on three elements collapse when looked at as permutation clones.

Neoliberalism and local consistency

Tomas Nagy
Jagiellonian University

We discuss the local consistency notion of strict width for certain ω-categorical relational structures. This property is known to have an algebraic characterization in the form of so-called quasi near-unanimity polymorphism. We show that for certain relational structures under consideration, having bounded strict width has a concrete consequence on their expressive power called implicational simplicity. This in turn yields an explicit bound on the relational width of the Constraint Satisfaction Problems (CSPs) over such relational structures, i.e., we obtain a tight bound on the amount of local consistency needed to ensure the satisfiability of any instance of such CSP. Our result applies to first-order expansions of any homogeneous k-uniform hypergraph, but more generally to any omega-categorical relational structure under the assumption of finite duality and general abstract conditions mainly on its automorphism group.

This is a joint work with Michael Pinsker.

Finitely Generated Varieties of Commutative BCK-algebras: Covers

Václav Cenker
Palacký University Olomouc

In the talk, we aims at describing all covers of any finitely generated variety of cBCK-algebras. It is known that subdirectly irreducible cBCK-algebras are rooted trees (concerning their order). Also, all subdirectly irreducible members of finitely generated variety are subalgebras of subdirectly irreducible generators of that variety. The first part of the talk focuses on subalgebras of finite subdirectly irreducible cBCK-algebras. In the second part of the talk, a construction is presented that provides all the covers of any finitely generated variety.

Incidence Matrix and Wiener Index with Python Code of Zero Divisor Graphs

Vijay Kumar Bhat
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The notion of the zero divisor graph (ZDG) of rings has been of interest since the recent past. In computer modelling, matrices are better accessible than graphs, and therefore, the representation of graphs by matrices is worth studying. To characterize molecular networks and establish the relationship between their structure and properties, topological indices are commonly used. In this article, we discuss the incidence matrix, code derived from the incidence matrix, and one topological index (i.e., Wiener index) of ZDG for $\mathbb{Z}_n$, $\mathbb{Z}_n[i]$, and Cartesian product $\mathbb{Z}_p \times \mathbb{Z}_q$ of finite fields $\mathbb{Z}_p$ and $\mathbb{Z}_q$.

Endomorphisms of semigroups of monotone transformations

Vítor Hugo Fernandes
NOVA Math

For $n \in \mathbb{N}$, let Ω_n be a finite set with n elements. Denote by PT_n the semigroup (under composition) of all partial transformations of Ω_n. The subsemigroup of PT_n of all full transformations of Ω_n and the (inverse) subsemigroup of all partial permutations (i.e. partial injective transformations) of Ω_n are denoted by T_n and I_n, respectively. In what follows, we suppose that Ω_n is a finite chain with n elements, e.g. $\Omega_n = \{1 < 2 < \cdots < n\}$. We say that a transformation s in PT_n is *order-preserving* [*order-reversing*] if $x \leqslant y$ implies $xs \leqslant ys$ [$xs \geqslant ys$], for all $x, y \in \mathrm{Dom}(s)$. A transformation that is either order-preserving or order-reversing is also called *monotone*. Notice that, the product of two order-preserving transformations or of two order-reversing transformations is order-preserving and the product of an order-preserving transformation by an order-reversing transformation is order-reversing. Moreover, the product of two monotone transformations is monotone. Denote by PO_n [PM_n] the subsemigroup of PT_n of all partial order-preserving [monotone] transformations of Ω_n, by O_n [M_n] the semigroup $PO_n \cap T_n$ [$PM_n \cap T_n$] of all full transformations of Ω_n that preserve the order [are monotone] and by POI_n [PMI_n] the inverse semigroup $PO_n \cap I_n$ [$PM_n \cap I_n$] of all partial order-preserving [monotone] permutations of Ω_n. In this presentation, we describe the monoids of endomorphisms of the semigroups PM_n, M_n and PMI_n of monotone transformations, as well as the monoids of endomorphisms of the semigroups PO_n, O_n and POI_n of order-preserving transformations.

These results were obtained in collaboration with several co-authors.

Practical methods for the study of amalgamation in certain quasivarieties

Wesley Fussner
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We give a range of techniques for studying the relative congruence extension property and amalgamation property in certain quasivarieties. In particular, we show that if Q is a congruence-distributive quasivariety, then Q has the relative congruence extension property if and only if its associated class of relatively finitely subdirectly irreducible members does. Further, we show that if Q is a quasivariety with the relative congruence extension property and the class of relatively finitely subdirectly irreducible members of Q is closed under subalgebras, then Q has the amalgamation property if and only if its class of relatively finitely subdirectly members has a one-sided amalgamation property. We also show that, under appropriate hypotheses, there are effective procedures for deciding whether a congruence-distributive variety has the congruence extension property and the amalgamation property.

The weak endomorphism semigroup of a partial equivalence

Yurii Zhuchok
JKU Linz
Luhansk Taras Shevchenko National University

A transformation f of a nonempty set X is called an *endomorphism* of a relation $\rho \subseteq X \times X$ if the condition $(a, b) \in \rho$ implies $(af, bf) \in \rho$ for all $a, b \in X$. The set of all endomorphisms of ρ is a semigroup with respect to the composition of transformations. This semigroup is called the *endomorphism semigroup* of ρ and it is denoted by $\mathrm{End}(X, \rho)$. A transformation f of X is called a *weak endomorphism* of $\rho \subseteq X \times X$ if $(a, b) \in \rho$ implies that $af = bf$ or $(af, bf) \in \rho$ for all $a, b \in X$ (see, e.g., [1]). The set of all weak endomorphisms of $p \subseteq X \times X$ with the operation of the composition of transformations is a semigroup and it is denoted by $\mathrm{WEnd}(X, \rho)$. It is clear that $\mathrm{End}(X, \rho)$ is a subsemigroup of $\mathrm{WEnd}(X, \rho)$. A binary relation on a nonempty set is called a *partial equivalence relation* if it is symmetric and transitive. In this talk we study the weak endomorphism semigroup of a partial equivalence relation. In particular, we find all weak (idempotent) endomorphisms of a partial equivalence and the regularity and coregularity conditions of the weak endomorphism semigroups of a partial equivalence. In terms of the wreath product of a symmetric transformation semigroup with a small category, we describe the faithful representation of the weak endomorphism semigroup of a partial equivalence relation. Also, we consider other properties of the weak endomorphism semigroup of a partial equivalence.

This is joint work with Olena Toichkina.

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Identifying Tractable Quantified Temporal Constraints within Ord-Horn

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The constraint satisfaction problem, parameterized by a relational structure, provides a general framework for expressing computational decision problems. An important class of templates used in this context are temporal structures, i.e., structures first-order definable over $(\mathbb{Q};<)$. In the standard setting, which allows only existential quantification over input variables, the complexity of temporal constraints has been fully classified. In the quantified setting, i.e., when one also allows universal quantifiers, there is only a handful of partial classification results and many concrete cases of unknown complexity. In this talk we present a significant progress towards understanding the complexity of the quantified constraint satisfaction problem (QCSP) for temporal structures. We provide a complexity dichotomy for quantified constraints over the Ord-Horn fragment, showing that all problems under consideration are in P or coNP-hard. In particular, we determine the complexity of QCSP$(\mathbb{Q}; x = y \Rightarrow x \geq z)$, hereby settling a question open for more than ten years.

This is joint work with Jakub Rydval and Michał Wrona.

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