Abstract:

We address Kreisel’s question “What more do we know if we have proved a theorem with restricted means, rather than only knowing that it is true?” Consider a formula $A$ posing a computational problem (in the sense of Kolmogorov 1932), for instance “there are infinitely many primes”. Then the simple answer to Kreisel’s question is that if we have a constructive proof of $A$, then this proof implicitly contains an algorithm to solve it. We discuss an example where a rather unexpected efficient algorithm (normalization by evaluation) is the computational content of Tait’s proof of the existence of normal forms in typed lambda-calculus.