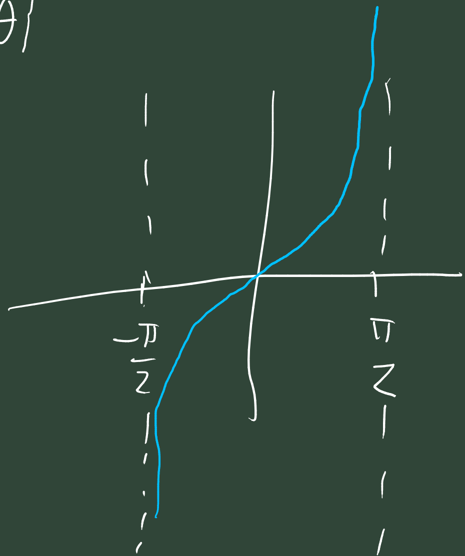
$$\int \frac{dx}{(x^2+1)^2} = \left| \begin{array}{l} \text{sub. (2)} \\ \underset{\in \mathbb{R}}{x} = \tan \underset{\in (-\frac{\pi}{2}, \frac{\pi}{2})}{\theta} \\ \theta = \arctan x \\ dx = \underbrace{(1+\tan^2\theta)}_{>0} d\theta \end{array} \right| =$$



$$= \int \frac{(1+\tan^2\theta)\, d\theta}{(\tan^2\theta + 1)^2} =$$

$$= \int \frac{d\theta}{1+\tan^2\theta} = \int \cos^2\theta \, d\theta =$$

$$= \int \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right) d\theta = \frac{\theta}{2} + \frac{1}{4}\sin 2\theta + C$$

$$= \frac{1}{2}\arctan x + \frac{1}{2}\,\frac{x}{1+x^2} + C$$

na $\mathbb{R}$

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\tan\theta\cos^2\theta =$$

$$= \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\int \frac{dx}{x^4+1} = \int \frac{?}{x^2+Ax+B}\,dx + \int \frac{?}{x^2+Cx+D}\,dx$$

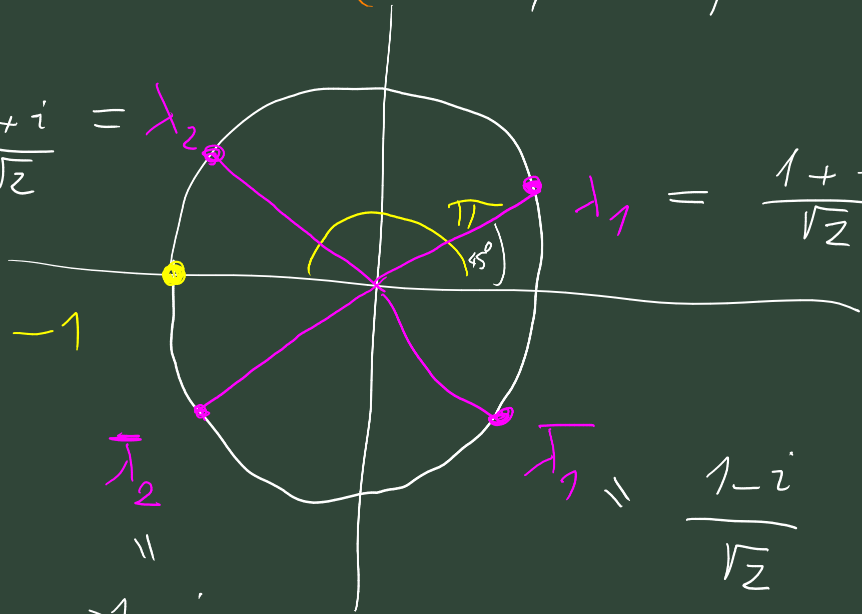
$$x^4+1 = \left(x^2+Ax+B\right)\left(x^2+Cx+D\right)$$

hledám x tak, že $x^4 = -1$ v $\mathbb{C}$

$$-1 = \cos\pi + i\sin\pi$$

$$x_k = 1\left(\cos\left(\frac{\pi+2k\pi}{4}\right) + i\sin\left(\frac{\pi+2k\pi}{4}\right)\right)$$

$$\frac{-1+i}{\sqrt{2}} = \lambda_2 \qquad \lambda_1 = \frac{1+i}{\sqrt{2}}$$



$$\bar{\lambda}_2 = \frac{-1-i}{\sqrt{2}} \qquad \bar{\lambda}_1 = \frac{1-i}{\sqrt{2}}$$

$$x^4+1 = (x-\lambda_1)(x-\bar{\lambda}_1)(x-\lambda_2)(x-\bar{\lambda}_2) =$$

$$= (x^2 - \overbrace{2\operatorname{Re}\lambda_1}^{\frac{1}{\sqrt{2}}} x + \overbrace{|\lambda_1|^2}^{1}) \cdot$$

$$\cdot (x^2 - \underbrace{2\operatorname{Re}\lambda_2}_{-\frac{1}{\sqrt{2}}} x + \underbrace{|\lambda_2|^2}_{1})$$

$$= (x^2 - \sqrt{2}x + 1)(x^2 + \sqrt{2}x + 1)$$

$$\int \frac{dx}{x^4 + 1} = \int \frac{Ex + F}{x^2 + \sqrt{2}x + 1}\,dx + \int \frac{Gx + H}{x^2 - \sqrt{2}x + 1}\,dx$$

$$= \ldots$$

$$\int R(\sin\theta, \cos\theta)\, d\theta$$

$\uparrow$ rac. funkce

1) sub. (2)

$$\tan\frac{\theta}{2} = t \quad \Longleftrightarrow \quad \theta = 2\arctan t$$

Speciální sub.

2) $R(\cdot,\cdot)$ je sudé v $\sin\theta$ i v $\cos\theta$

$$R(\sin\theta, \cos\theta) = R(-\sin\theta, \cos\theta)$$

$$R(\sin\theta, \cos\theta) = R(\sin\theta, -\cos\theta)$$

sub. (1)

$$\tan\theta = t$$

$$R(\sin\theta, \cos\theta) = \tilde{R}(\underbrace{\sin^2\theta}_{1-\cos^2\theta}, \cos^2\theta)$$

$$= \tilde{\tilde{R}}(\underbrace{\cos^2\theta}_{\frac{1}{1+\tan^2\theta}}) = \frac{\tilde{\tilde{\tilde{R}}}(\tan\theta)}{\cos^2\theta}$$

Pr.

$$\int \frac{\sin^2 x}{1+\sin^2 x}\,dx = \int \frac{1-\cos^2 x}{2-\cos^2 x}\,dx =$$

$$= \int \frac{1 - \frac{1}{1+\tan^2 x}}{2 - \frac{1}{1+\tan^2 x}} \cdot \frac{\frac{1}{1+\tan^2 x}}{\cos^2 x}\,dx$$

$$= \int \frac{1 - \frac{1}{1+\tan^2 x}}{2 - \frac{1}{1+\tan^2 x}} \cdot \frac{\frac{1}{1+\tan^2 x}}{\cos^2 x} \, dx$$

$$= \int \frac{(1+\tan^2 x) - 1}{1 + 2\tan^2 x} \, \frac{1}{1+\tan^2 x} \, \frac{dx}{\cos^2 x} =$$

$$= \left| \begin{array}{c} \text{sub. (1)} \\ \tan x = t \\ \frac{dx}{\cos^2 x} = dt \end{array} \right| = \int \frac{t^2 \, dt}{(1+2t^2)(1+t^2)} =$$

$$= \ldots$$

3) R je liché v $\sin x$

$\Rightarrow$ $\cos x = t$ (sub. (1))

4) R je liché v $\cos x$

$\Rightarrow$ $\sin x = t$ (sub. (1))

Př.

$$\int \frac{\sin x \cos x}{1+\sin^3 x}\,dx = \left| \begin{array}{c} \text{sub.} \\ \sin x = t \\ \cos x\,dx = dt \end{array} \right| =$$

$$= \int \frac{t}{1+t^3}\,dt$$

sub zu e^x, $\ln x$

Pr.

$$\int \frac{e^x\,dx}{e^x(e^{2x}+e^x-2)} = \left| \begin{array}{c} \text{sub. (1)} \\ e^x = t \\ e^x dx = dt \end{array} \right| =$$

$$= \int \frac{dt}{t(t^2+t-2)} = \int \left(\frac{A}{t} + \frac{B}{t-1} + \frac{C}{t+2} \right) dt$$

$-1 \quad 2$

Pr.

$$\int \frac{dx}{x(\ln^3 x+1)} = \left| \begin{array}{c} \text{sub. (1)} \\ \ln x = t \\ \frac{dx}{x} = dt \end{array} \right| =$$

$$= \int \frac{dt}{t^3+1} = \dots$$

$$\sinh^2 x = \frac{\cosh 2x - 1}{2}$$

$$\cosh^2 x = \frac{\cosh 2x + 1}{2}$$

Pr.

$$\int \frac{x^2}{\sqrt{x^2+1}}\,dx = \left| \begin{array}{l} \text{sub. } (2) \\ x = \sinh\theta \\ dx = \cosh\theta\, d\theta \\ \theta = \operatorname{argsinh} x \end{array} \right| =$$

$$= \int \frac{\sinh^2\theta}{\cancel{\cosh\theta}} \cancel{\cosh\theta}\, d\theta = \int \sinh^2\theta\, d\theta =$$

$2\sinh\theta\cosh\theta$

$$= \int \frac{\cosh 2\theta - 1}{2}\, d\theta = \frac{\sinh 2\theta}{4} - \frac{\theta}{2} + C$$

$$= \frac{1}{2}\sinh\theta \boxed{\sqrt{1+\sinh^2\theta}} - \frac{\theta}{2} + C$$

$$= \frac{1}{2} \sinh\theta \sqrt{1+\sinh^2\theta} - \frac{\theta}{2} + C$$

$$= \frac{x}{2}\sqrt{1+x^2} - \frac{1}{2}\operatorname{argsinh} x + C$$

na $\mathbb{R}$

Limity funkcí podruhé

$$\lim_{x\to\infty} \frac{a_n x^n + \dots + a_1 x + a_0}{A_m x^m + \dots + A_1 x + A_0} = ?$$

$a_k \in \mathbb{R}, \quad a_n \neq 0, \quad n \in \mathbb{N}$

$A_k \in \mathbb{R}, \quad A_m \neq 0, \quad m \in \mathbb{N}$

$$\lim_{x\to\infty} \frac{a_n x^n + \dots + a_1 x + a_0}{A_m x^m + \dots + A_1 x + A_0} =$$

$$= \lim_{x\to\infty} \frac{x^n}{x^m} \cdot \frac{a_n + \frac{a_{n-1}}{x} + \dots + \frac{a_0}{x^n}}{A_m + \frac{A_{m-1}}{x} + \dots + \frac{A_0}{x^m}}$$

$$= \lim_{x\to\infty} \frac{x^n}{x^m} \cdot \frac{a_n + \frac{a_{n-1}}{x} + \dots + \frac{a_0}{x^n}}{A_m + \frac{A_{m-1}}{x} + \dots + \frac{A_0}{x^m}}$$

Vim

$$\lim_{x\to\infty} \frac{1}{x^n} = 0 \quad , \quad n > 0$$

$\Rightarrow$

AL(?)

$$= \lim_{x\to\infty} x^{n-m} \left(\frac{a_n + \underbrace{\lim_{x\to\infty} \frac{a_{n-1}}{x}}_{=0} + \dots + \underbrace{\lim_{x\to\infty} \frac{a_0}{x^n}}_{=0}}{A_m + \underbrace{\lim_{x\to\infty} \frac{A_{m-1}}{x}}_{=0} + \dots + \underbrace{\lim_{x\to\infty} \frac{A_0}{x^m}}_{=0}} \right)$$

$$= \frac{a_n}{A_m} \lim_{x\to\infty} x^{n-m}$$

$$\frac{a_n}{A_m} \lim_{x\to\infty} x^{n-m} = \begin{cases} \frac{a_n}{A_m} \cdot (+\infty), & n>m \\ \frac{a_n}{A_m}, & n=m \\ 0, & n<m \end{cases}$$

$$\lim_{x\to\infty} x^n = +\infty, \quad n>0$$