$$\min\{\sup A, \sup B\} \geq \sup(A \cap B)$$

$$(\cancel{\leq})$$

Rozdělíme 2 případy

$\cancel{\text{i)} \quad A \subseteq B}$

$\cancel{\text{ii)} \quad B \subseteq A}$

$A = \{1; 3\}$

$B = \{1; 2\}$

$$\sqrt[3]{(a+1)(b+1)(c+1)} \leq \frac{a+b+c+3}{3}$$

AG

1) $$8 = (a+1)(b+1)(c+1) \leq \left(\frac{a+b+c+3}{3}\right)^3$$

$$2 \leq \frac{a+b+c+3}{3} \quad \Big| \cdot 3$$

$$6 \leq a+b+c+3$$

$$3 \leq a+b+c$$

$$\boxed{1 \underset{\geq}{\leq} \frac{a+b+c}{3}}$$

$$\sqrt[3]{abc} \leq \frac{a+b+c}{3}$$

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$$\lim_{x \to 1^+} \lfloor x \rfloor = 1 \quad \ldots$$

$$\lim_{x \to 1} \frac{x^2-1}{\underbrace{2x^2-x-1}_{(x-1)(2x+1)}} \quad \ldots$$

$$\lim_{x\to 0} \frac{(1+mx)^n - (1+nx)^m}{x^2} = ?$$

kde $\boxed{n, m \in \mathbb{N}}$

$$(1+mx)^n = \sum_{k=0}^{n} \binom{n}{k} m^k x^k \, 1^{n-k}$$

$$= \underline{1} + \underline{\binom{n}{1} mx} + \binom{n}{2} m^2 x^2 + \dots + \binom{n}{n} m^n x^n$$

$n$

$$(1+nx)^m = \underline{1} + \underline{\binom{m}{1} nx} + \binom{m}{2} n^2 x^2 + \dots + \binom{m}{m} n^m x^m$$

$m$

$$\to \quad = \lim_{x\to 0} \frac{\binom{n}{2} m^2 x^2 - \binom{m}{2} n^2 x^2 + x^3 B_{n,m}(x)}{x^2}$$

polynom

$$= \lim_{x \to 0} \left[ \binom{n}{2} m^2 - \binom{m}{2} n^2 + x B_{n,m}(x) \right]$$

$$= \binom{n}{2} m^2 - \binom{m}{2} n^2 + \underbrace{0 B_{n,m}(0)}_{0}$$

$$= \frac{1}{2} \left[ n(n-1) m^2 - m(m-1) n^2 \right]$$

$$= \frac{nm}{2} \left[ \overset{nm - m}{(n-1)m} - \overset{-nm + n}{n(m-1)} \right]$$

$$= \frac{1}{2} nm (n - m)$$

18. $a \in [0, +\infty)$

$$\boxed{\lim_{x \to a^+} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2 - a^2}}}$$

$$\sqrt{x-a}\,\sqrt{x+a}$$

$$\downarrow$$

$$\cdot \frac{\sqrt{x^2-a^2}}{\sqrt{x^2-a^2}}$$

$$= \lim_{x \to a^+} \frac{(\sqrt{x}-\sqrt{a})\sqrt{x^2-a^2} + (x-a)\sqrt{x+a}}{x^2-a^2} =$$

$(x-a)(x+a)$

(AL)?

$$= \lim_{x \to a^+} \frac{(\sqrt{x}-\sqrt{a})\sqrt{x^2-a^2}}{x^2-a^2} + \lim_{x \to a^+} \frac{\sqrt{x+a}}{x+a}$$

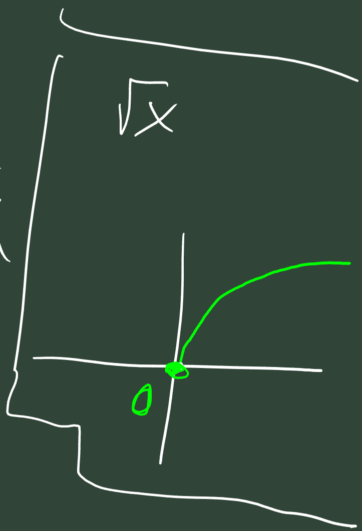
$$= \lim_{x \to a^+} \frac{(\sqrt{x}-\sqrt{a})\sqrt{x^2-a^2}}{\left(\sqrt{x^2-a^2}\right)^2} + \frac{\sqrt{a+a}}{a+a} =$$

$$= \lim_{x \to a^+} \frac{\sqrt{x} - \sqrt{a}}{\sqrt{x^2 - a^2}} + \frac{1}{\sqrt{2a}} =$$

$$= \lim_{x \to a^+} \frac{(\sqrt{x} - \sqrt{a})(\sqrt{x} + \sqrt{a})}{\sqrt{x^2 - a^2}\,(\sqrt{x} + \sqrt{a})} + \frac{1}{\sqrt{2a}}$$

$$= \lim_{x \to a^+} \frac{(\sqrt{x - a})^2}{\sqrt{(x-a)(x+a)}\,(\sqrt{x} + \sqrt{a})} + \frac{1}{\sqrt{2a}}$$

$$= \lim_{x \to a^+} \frac{\sqrt{x - a}}{\sqrt{x+a}\,(\sqrt{x} + \sqrt{a})} + \frac{1}{\sqrt{2a}}$$




$$= 0 + \frac{1}{\sqrt{2a}} = \underline{\underline{\frac{1}{\sqrt{2a}}}} = (2a)^{-1/2}$$

$$8 = (1+a)(1+b)(1+c)$$

$a,b,c \geq 0$

$$1 = \left(\frac{1+a}{2}\right)\left(\frac{1+b}{2}\right)\left(\frac{1+c}{2}\right) \geq$$

$$\geq \sqrt{1\cdot a}\ \sqrt{1\cdot b}\ \sqrt{1\cdot c} = \sqrt{abc} \quad \Big|\ ^2$$

$$1 \geq a\cdot b\, c$$

$\square$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

2.

$$\lim_{x\to 0} \frac{\sqrt{1-\cos x^2}}{1-\cos x}\cdot\frac{\sqrt{1+\cos x^2}}{\sqrt{1+\cos x^2}} =$$

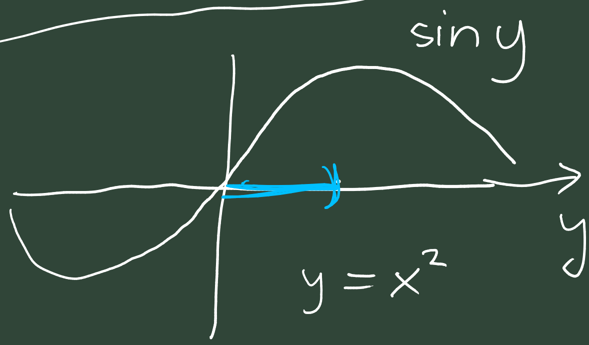
AL

$$= \lim_{x\to 0}\frac{\sqrt{1-\cos^2 x^2}}{1-\cos x}\;\underbrace{\lim_{x\to 0}\frac{1}{\sqrt{1+\cos x^2}}}_{\frac{1}{\sqrt{2}}} =$$

$$= \frac{1}{\sqrt{2}}\lim_{x\to 0}\frac{\sqrt{\sin^2 x^2}}{1-\cos x}$$

$$= \frac{1}{\sqrt{2}}\lim_{x\to 0}\frac{\sin x^2}{1-\cos x} =$$

$$= \frac{1}{\sqrt{2}}\lim_{x\to 0}\frac{\sin x^2}{x^2}\cdot\frac{x^2}{1-\cos x}$$




$\sin x^2 > 0$
pro $x$ malé
(blízko 0)

$$\frac{1}{\sqrt{2}} \lim_{x\to 0} \frac{\sin x^2}{x^2} \cdot \frac{x^2}{1-\cos x} \overset{AL(?)}{=}$$

$$\overset{AL(?)}{=} \frac{\dfrac{1}{\sqrt{2}} \lim\limits_{x\to 0} \dfrac{\sin x^2}{x^2}}{\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}} =$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} \longrightarrow \frac{1}{2}$$

$$= \frac{2}{\sqrt{2}} \lim_{x\to 0} \frac{\sin x^2}{x^2} =$$

$$= \sqrt{2} \lim_{y\to 0} \frac{\sin y}{y}$$

$$= \sqrt{2}$$

VOLSF (P)

$\varphi(x) = x^2$

$f(y) = \dfrac{\sin y}{y}$

$\varphi(x) \neq 0$

na $P(0, \Delta)$

$x^2 \neq 0$ na $\mathbb{R} \setminus \{0\}$

$$\frac{2}{\sqrt{2}} = \sqrt{2}$$

3. $$\lim_{x \to 0} \frac{\operatorname{tg} x - \sin x}{x^3} =$$

$$= \lim_{x \to 0} \frac{\frac{\sin x}{\cos x} - \sin x}{x^3}$$

$$= \lim_{x \to 0} \frac{\sin x - \cos x \sin x}{x^3 \cos x}$$

AL

$$= \lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{\frac{1}{\cos x} - 1}{x^2}$$

$$\underbrace{\lim_{x\to 0}\frac{\sin x}{x}}_{=1} \qquad \lim_{x\to 0}\frac{\frac{1}{\cos x}-1}{x^2} =$$

$$= \lim_{x\to 0}\frac{1-\cos x}{x^2\cos x} \overset{AL}{=} \overbrace{\lim_{x\to 0}\frac{1}{\cos x}}^{1\ (\text{ze spojitosti})} \cdot$$

$$\cdot \underbrace{\lim_{x\to 0}\frac{1-\cos x}{x^2}}_{=\frac{1}{2}\ (\text{známá limita})} = \underline{\underline{\frac{1}{2}}}$$

$$\lim_{x\to\pi} \frac{\sin nx}{\sin mx} = ? \quad , \quad n,m \in \mathbb{N}$$

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

PŘESTÁVKA

POKRAČOVÁNÍ 14:35

$$y = x - \pi \iff x = \pi + y$$

= | VOLSF (P)

$\varphi(x) = x - \pi$

$$f(y) = \frac{\sin n(\pi + y)}{\sin m(\pi + y)}$$

$\lim_{x\to\pi} \varphi(x) = 0$

$\varphi(x) \neq 0$ na prst. okolí $x \in P(\pi, \Delta)$ |

$$= \lim_{y\to 0} \frac{\sin n(\pi+y)}{\sin m(\pi+y)}$$

$$= \lim_{y \to 0} \frac{\sin n(\pi + y)}{\sin m(\pi + y)} =$$

sin je $\pi$-antiperiod.
tj: $\forall \theta \in \mathbb{R}$:
$\sin(\theta + \pi) = -\sin\theta$
$\cos(\theta + \pi) = -\cos\theta$

$$= \lim_{y \to 0} \frac{(-1)^n \sin ny}{(-1)^m \sin my} =$$

$$\boxed{\begin{array}{c} 1, -1 \\ x = \frac{1}{x} \\ (-1)^m = (-1)^{-m} \end{array}}$$

$$= (-1)^{n\cancel{-}m} \lim_{y \to 0} \frac{\sin ny}{ny} \cdot \frac{my}{\sin my} \cdot \frac{n}{m}$$

$$\overset{AL}{=} (-1)^{n+m} \left( \lim_{y \to 0} \frac{\sin ny}{ny} \right) \Big/ \left( \lim_{y \to 0} \frac{\sin my}{my} \right) \frac{n}{m}$$

$$\lim_{y\to 0}\frac{\sin ny}{ny} = \left|\begin{array}{l} \text{VOLSF (P)} \\ z = ny \\ z\to 0,\ z\neq 0 \\ \qquad\quad \text{pro } y\neq 0 \end{array}\right| =$$

$$= \lim_{z\to 0}\frac{\sin z}{z} = 1$$

$$= (-1)^{n+m}\,\frac{1}{1}\cdot\frac{n}{m}$$

$$= (-1)^{n+m}\,\frac{n}{m}$$

$$(-1)^{n-m} = (-1)^{n-m}\cdot(-1)^{2m} = (-1)^{n+m}$$

18) $$\lim_{x \to 0^+} (1+x)^{\frac{1}{x}}$$

$$\boxed{\exp x \equiv e^x}$$

$$f(x)^{g(x)} = \exp\left( [\ln f(x)] \cdot g(x) \right)$$

$$\lim_{x \to 0^+} x^0 = 1$$

$$\neq$$

$$\lim_{x \to 0^+} 0^x = 0$$

$$\lim_{x \to 0^+} \frac{1}{x} = +\infty$$

$$\lim_{x \to 0^+} (1+x) = 1$$

$$\lim_{x \to 0} (1+x)^{1/x} = \lim_{x \to 0} \exp\left[ \frac{\ln(1+x)}{x} \right]$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$$ (známá limita)

$$\Updownarrow$$

$$\lim_{x \to 1} \frac{\ln x}{x-1} = 1$$

VOLSF (S) $\Rightarrow$ $= e$

$f(y) = e^y$ ... spojitá

$g(x) = \frac{\ln(1+x)}{x}$

vím $\lim_{x \to 0} g(x) = 1$

$$\Rightarrow \lim_{x \to 0} f(g(x)) = f(1) = e$$

$$\lim_{x \to \frac{\pi}{2}} (\sin x)^{\operatorname{tg} x} =$$

$$= \lim_{x \to \frac{\pi}{2}} \exp \left[ \ln(\sin x) \operatorname{tg} x \right] =$$

$$= \left| \begin{array}{l} \text{VOLSF (P)} \\ y = x - \frac{\pi}{2} \\ x = y + \frac{\pi}{2} \end{array} \right| = \lim_{y \to 0} \exp \left[ \ln\left(\sin\left(y + \frac{\pi}{2}\right)\right) \cdot \right.$$

$$\left. \cdot \operatorname{tg}(y + \pi) \right]$$

$$\lim_{y\to 0} \ln\left(\sin\left(y+\frac{\pi}{2}\right)\right) \operatorname{tg}\left(y+\frac{\pi}{2}\right) =$$

$$\sin\left(y+\frac{\pi}{2}\right) = \cos(-y) = \cos y$$

$$\operatorname{tg}\left(y+\frac{\pi}{2}\right) = \frac{\sin\left(y+\frac{\pi}{2}\right)}{\cos\left(y+\frac{\pi}{2}\right)} = \frac{\cos y}{\sin(-y)} = -\frac{\cos y}{\sin y}$$

$$= -\operatorname{cotg} y = -\frac{1}{\operatorname{tg} y}$$

$$= -\lim_{y\to 0} \ln(\cos y) \frac{1}{\operatorname{tg} y}$$

$$= \lim_{y\to 0} \ln(\cos y) \frac{-1}{\operatorname{tg} y} =$$

$$= -\lim_{y\to 0} \underbrace{\frac{\ln(\cos y)}{\cos y - 1}}_{\substack{\downarrow\ \cos y \to 1 \\ 1}} \cdot \underbrace{\frac{\cos y - 1}{y^2}}_{\substack{\downarrow \\ \frac{1}{2}}} \cdot \underbrace{\frac{y}{\operatorname{tg} y}}_{\substack{\downarrow \\ 1}} \cdot \underbrace{y}_{\substack{\downarrow \\ 0}}$$

známé limity

$$\frac{\ln x}{x-1} \to 1$$

pro $x \to 1$

$$\overset{AL}{=} -1 \cdot \frac{1}{2} \cdot 1 \cdot 0 = 0$$

+ VOLSF (P)

$\Rightarrow$ výsledok je $e^0 = 1$

$$\begin{pmatrix} \text{VOLSF} \\ (S) \end{pmatrix}$$