NMSA409, NMFP403 Stochastic processes 2
Expected learning outcomes

Jiří Dvořák

Version 7. 8. 2026

This document summarizes the expected learning outcomes for the course NMSA409, NMFP403 Stochastic processes 2, taught in the Winter semester 2026/27 at the Faculty of Mathematics and Physics, Charles University, Prague. At present, the document applies only to the lectures and not to the exercise classes.

The expected learning outcomes describe the skills and competencies that students should possess after mastering the material covered in a given section. They should be concise, specific, and measurable. If you find that any of them do not satisfy these criteria, please let me know!

The purpose of formulating these expected learning outcomes explicitly is to help students verify that they have truly mastered the material and to encourage reflection on their own learning process. These do not, however, constitute a list of questions for the exam.

1 Background on Stochastic Processes

After completing this chapter, the student is able to:

- describe the concept of a stochastic process, its state space, trajectories, and finite-dimensional distributions,
- formulate and explain the Daniell–Kolmogorov theorem and its significance for process construction,
- identify key properties of Gaussian processes and check whether a given process is Gaussian,
- explain how complex-valued random variables and processes are defined,
- define independent-increment processes and provide examples,
- describe the defining properties of Poisson and Wiener processes.

2 Autocovariance Function and Stationarity

After completing this chapter, the student is able to:

- define the autocovariance and autocorrelation functions of a stochastic process,
- compute autocovariance and autocorrelation functions for different models,
- describe the properties of autocovariance functions, including the equivalent characterization,
- decide whether a given function is the autocovariance function of some stochastic process,
- explain the relationship between strict, weak, and covariance stationarity,
- provide examples of stochastic processes which are (are not) strictly, weakly, or covariance stationary,
- decide, for various models of stochastic processes, whether they are strictly, weakly or covariance stationary,
- analyze the implications of stationarity assumptions in modelling of stochastic processes.

3 Space $L^2(\Omega, \mathcal{A}, \mathbb{P})$

After completing this chapter, the student is able to:

- define an inner product space, norm, and Hilbert space,
- explain the construction of the space $L^2(\Omega, \mathcal{A}, \mathbb{P})$,
- define and interpret mean square convergence of random variables,
- describe the construction of the Hilbert space generated by a stochastic process and explain the difference between situations where the index set T is finite or infinite,
- define and characterize the convergence of stochastic processes in $L^2(\Omega, \mathcal{A}, \mathbb{P})$,
- explain how the first and second moments behave under mean square convergence.

4 Mean Square Continuity

After completing this chapter, the student is able to:

- define mean square continuity of a stochastic process,
- relate mean square continuity to continuity of the autocovariance function,
- provide examples of stochastic processes which are (are not) mean square continuous,
- determine whether a stochastic process is mean square continuous,
- distinguish mean square continuity from continuity of sample trajectories.

5 Spectral Decomposition of Autocovariance Functions

After completing this chapter, the student is able to:

- explain the spectral representation of an autocovariance function,
- define the spectral distribution function and spectral density,
- find the spectral distribution function and/or spectral density of various weakly stationary processes,
- determine whether a spectral density exists for a given autocovariance function.

6 Spectral Representation of a Stochastic Process

After completing this chapter, the student is able to:

- define an orthogonal increment process,
- compare the orthogonal increment property to independent increments,
- describe the construction of stochastic integrals with respect to orthogonal increment processes,
- explain the isometry property of stochastic integrals,
- formulate and apply the theorems about the spectral representation of weakly stationary processes,
- describe the relationship between spectral distribution functions and orthogonal distribution functions.

7 Linear Models

After completing this chapter, the student is able to:

- describe the properties of white-noise sequences, including the spectral distribution function and spectral density,
- define MA, AR, and ARMA models,
- interpret the role of characteristic polynomials in determining their stationarity,
- compute autocovariance functions of linear time-series models,
- find spectral densities of MA, AR, and ARMA sequences,
- explain the concepts of causality and invertibility,
- suggest two different problems that can be solved using Yule–Walker equations,
- analyze the effect of linear filters on stochastic processes,
- explain the role of the transfer function in the construction of linear filters.

8 Ergodicity

After completing this chapter, the student is able to:

- define mean square ergodicity for random sequences,
- explain the connection between ergodicity and the law of large numbers,
- relate ergodicity to properties of autocovariance functions,
- determine whether a stationary sequence is mean square ergodic,
- provide examples of random sequences which are (are not) mean square ergodic.

9 Prediction in the Time Domain

After completing this chapter, the student is able to:

- formulate the prediction problem,
- explain how it can be solved using orthogonal projections,

- construct best linear predictions based on finite number of observations,
- construct best linear predictions based on infinite history,
- compute prediction errors for weakly stationary time series,
- argue why the prediction error is a property of the method, not of a particular (numeric) prediction,
- derive prediction formulas for AR and ARMA sequences,
- explain the role of causality and invertibility in the construction of these predictions,
- formulate the problem of filtration of signal and noise and solve it using projections.

10 Partial Autocorrelation Function

After completing this chapter, the student is able to:

- define the partial autocorrelation function (PACF),
- interpret PACF in connection to the partial correlation coefficient,
- compute PACF values using the projection-based definition,
- link the PACF values to the values of the autocorrelation function,
- explain the relationship between PACF and AR model order,
- compare the behavior of ACF and PACF for AR and MA processes,
- use PACF estimates to support model identification.

11 Estimation of Moment Properties

After completing this chapter, the student is able to:

- estimate the mean of a weakly stationary time series,
- estimate the autocovariance, autocorrelation, and partial autocorrelation functions from observations,
- explain the rather non-standard choice of the normalization factor in the estimation of the autocovariance function.

12 Estimation of Parametric Models

After completing this chapter, the student is able to:

- estimate parameters of AR models using Yule–Walker equations, least-squares method, maximum likelihood method and conditional maximum likelihood method and compare these approaches,
- explain the advantages of the conditional maximum likelihood compared to the classical maximum likelihood,
- estimate parameters of MA models using moment-based methods,
- estimate parameters of ARMA models using a multistage procedure, combining the previous estimates,
- assess identifiability, causality, and invertibility constraints of the estimation methods.

13 Estimation of Spectral Density

After completing this chapter, the student is able to:

- define and compute the periodogram of a time series,
- explain the statistical properties and limitations of the periodogram,
- identify dominant frequencies in observed data,
- apply Fisher's test to detect periodic components,
- construct smoothed spectral density estimators using kernel methods,
- compare parametric and nonparametric spectral density estimation approaches,
- interpret estimated spectral densities in terms of underlying process dynamics.