2. Computations

2.1. Find all singularities of $f = y^2 + y - x^3 + x^2 - x + 1 \in \mathbb{F}_3[x, y]$ and compute the genus of the AFF over $\mathbb{F}_3$ given by $f(\alpha, \beta) = 0$.

10 points

2.2. Let $f = y^2 - (x^3 - x + 1) \in \mathbb{C}[x, y]$ and $A = 1P_{(1,1)} + 3P_{(0,1)} + 5P_{(-1,1)} - 8P_\infty$. Explain why A is a well-defined divisor and compute $\deg(A)$. Is A principal?

5 points

2.3. Let $w = y^2 - x^3 \in \mathbb{F}_7[x, y]$. Find an $\mathbb{F}_7$-isomorphism of fields $\mathbb{F}_7(z) \to \mathbb{F}_7(V_f)$.

5 points

3. Proofs

3.1. If L is an AFF over K, $P \in \mathbb{P}_{L/K}$, prove that $\mathcal{O}_P$ is a uniquely defined discrete valuation ring and that $\deg P$ is finite.

10 points

3.2. Formulate and proof the assertion characterizing eliptic Weierstrass equiation polynomials.

10 points