

1. Contours, open and closed sets

1. Determine contours and domains of the following functions.

$$\begin{aligned}f_1(x, y) &= x + \sqrt{y}, & f_2(x, y) &= \frac{y}{x}, & f_3(x, y) &= x^2 + y^2, \\f_4(x, y) &= x^2 - y^2, & f_5(x, y) &= \sqrt{xy}, \\f_6(x, y) &= \sqrt{1 - x^2 - y^2}, & f_7(x, y) &= \frac{1}{\sqrt{x^2 + y^2 - 1}}, \\f_8(x, y) &= \sqrt{(x^2 + y^2 - 1)(4 - x^2 - y^2)}, & f_9(x, y) &= \sqrt{\sin(x^2 + y^2)}, \\f_{10}(x, y) &= \operatorname{sgn}(\sin x \cdot \sin y), & f_{11}(x, y) &= |x| + y.\end{aligned}$$

2. Decide whether the following sets are open or closed, determine their interior, closure and boundary.

$$\begin{aligned}A_1 &= \mathbb{Q}, & A_2 &= \mathbb{N}, & A_3 &= \{1/n; n \in \mathbb{N}\}, \\A_4 &= \{[x, y] \in \mathbb{R}^2; x > 0, y \leq 0\}, & A_5 &= \{[x, y] \in \mathbb{R}^2; x^2 + y^2 < 1\}, \\A_6 &= \{[x, y] \in \mathbb{R}^2; x^2 + y^2 \geq 1\}, & A_7 &= \{[x, y] \in \mathbb{R}^2; x^2 + e^y > 17\}, \\A_8 &= \{[x, y] \in \mathbb{R}^2; x^2 + y^2 + 2xy = 5\}, \\A_9 &= \{[x, y, z] \in \mathbb{R}^3; x \geq 0, y > 0, x + y = 2, z \leq 0\}.\end{aligned}$$

3. Determine closure, boundary and interior of the set $M = \{(x, y) \in \mathbb{R}^2; |x + y| - x - y > 0\}$.