

Test B – 23. 1. 2012

Problem 1 (15 points). Compute the limit of the sequence.

$$\lim_{n \rightarrow \infty} \frac{(n!)^2 \left(\sqrt{n^2 + \sin \frac{1}{n}} - \sqrt{n^2 - \sin \frac{1}{n}} \right)}{(4^n + (n-1)! \sin n + n!) \cdot (n-2)!}$$

Problem 2 (15 points). Compute the limit of the function.

$$\lim_{x \rightarrow 0+} \left(\frac{1 + \sqrt{x} \cdot 4^x}{1 + \sqrt{x} \cdot 3^x} \right)^{\frac{\sqrt{e^x - \cos x}}{x^2}}$$

Problem 3 (15 points). Find all points, where the given function f is continuous (continuous from the left, continuous from the right), and compute its derivative (onesided derivatives) of the function at each point, where it exists.

$$f(x) = \operatorname{sgn}(x^3 - x) \cdot \sqrt[3]{(\log(x+2))^4}$$

Problem 4 (15 points). Investigate the behavior of the function

$$f(x) = x^2 e^{-|2x+1|}.$$

Answers

1. 1

2. $4/3$

3. The function f is continuous at each point of $(-2, \infty) \setminus \{0, 1\}$. The derivative exists at each point of $(-2, \infty)$, $f'(-1) = 0$, $f'(0) = -\infty$, $f'(1) = +\infty$, and

$$f'(x) = \frac{4}{3} \frac{\sqrt[3]{\log(x+2)}}{x+2} \cdot \operatorname{sgn}(x^3 - x), \quad x \in (-2, \infty) \setminus \{-1, 0, 1\}.$$

4.

