Test A – 9. 1. 2012

1. Compute the limit of the sequence
$$\lim_{n\to\infty} \frac{2^n \sqrt[3]{n^3 + n^2} - n\sqrt[3]{8^n + 1}}{\sqrt[n]{2n^2 + 1}}. \qquad \text{(15 points)}$$

2. Compute the limit of the function
$$\lim_{x\to 0} (2^{\cos x} - 1)^{\frac{1}{\arcsin^2 x}}. \qquad \text{(15 points)}$$

3. Find all points, where the given function f is continuous (continuous from the left, continuous from the right), and compute its derivative (onesided derivative) of the function at each point, where it exists.
$$f(x) = [x + 1]x^3. \qquad \text{(15 points)}$$

4. Investigate behavior of the function
$$f(x) = \sqrt[3]{x^2 + \frac{1}{x}}. \qquad \text{(15 points)}$$