4.4. Elementary functions.

Theorem 4.15. *There exists a unique function* **logarithm** *(*log*) with the following properties*

(L1) $D(\log) = (0, +\infty)$ *and* log *is increasing on* $(0, +\infty)$*,*
(L2) $\forall x, y \in (0, +\infty): \ \log xy = \log x + \log y$*,*
(L3) $\lim_{x\to 1} \frac{\log x}{x-1} = 1$.

Definition. **Exponential function** (exp) is defined as the inverse function to log.

Definition. Let $a, b \in \mathbf{R}$, $a > 0$. The number a^b is defined by $a^b = \exp(b \log a)$.

Theorem 4.16. *There exists a unique positive real number* π *and a unique function* **sine** *(*sin*) with the following properties*

(S1) $D(\sin) = \mathbf{R}$*,*
(S2) sin *is increasing on* $[-\pi/2, \ \pi/2]$*,*
(S3) $\sin 0 = 0$*,*
(S4) $\forall x, y \in \mathbf{R}: \ \sin(x + y) = \sin x \cdot \sin(\frac{\pi}{2} - y) + \sin(\frac{\pi}{2} - x) \cdot \sin y$*,*
(S5) $\lim_{x\to 0} \frac{\sin x}{x} = 1$.

Definition. The function **cosine** (cos) is defined by

$$\cos x = \sin\left(\frac{\pi}{2} - x\right), \quad x \in \mathbf{R}.$$

The function **tangent** (tan) is defined by

$$\tan x = \frac{\sin x}{\cos x}$$

for each real x, where the fraction makes sense, i.e.,

$$D(\tan) = \{x \in \mathbf{R};\ x \neq (2k + 1)\pi/2,\ k \in \mathbf{Z}\}.$$

The symbol cot stands for the function **cotangent**, which is defined on the set $D(\cot) = \{x \in \mathbf{R};\ x \neq k\pi, k \in \mathbf{Z}\}$ by

$$\cot x = \frac{\cos x}{\sin x}.$$

Definition (cyclometric functions)**.** We define functions **arcsine** (arcsin), **arccosine** (arccos), **arctangent** (arctan), **arccotangent** (arccot) by

$$\begin{aligned} \arcsin &= (\sin|_{[-\frac{\pi}{2},\frac{\pi}{2}]})^{-1}, \\ \arccos &= (\cos|_{[0,\pi]})^{-1}, \\ \arctan &= (\tan|_{(-\frac{\pi}{2},\frac{\pi}{2})})^{-1}, \\ \operatorname{arccot} &= (\cot|_{(0,\pi)})^{-1}. \end{aligned}$$

Theorem 4.17. *Functions* log, exp, sin, cos, tan, cot, arcsin, arccos, arctan *and* arccot *are continuous on their domains.*