1. GRAMMAR SCHOOL MATHEMATICS

1. Solve the equations in $\mathbf{R}$.

$$\frac{x-2}{2x-8} \geq 1, \qquad \log_{\frac{1}{3}}(x^2-3x+3) \geq 0, \qquad \frac{x+2}{x+3} > \frac{2x+3}{x+6}.$$

2. Draw a graph of the function $f(x) = \Big|\Big|\big||x|-1\big|-1\Big|-1\Big|$, $x \in \mathbf{R}$.

3. Solve the equations in $\mathbf{R}$.

$$\sin 2x = \sin x, \quad 2\sin x + \cos x = 1, \quad \log(x^2+1) = 2\log(3-x).$$

4. Express the function $\cos 5x$ a $\sin 5x$ using functions $\cos x$ a $\sin x$.

5. Solve the inequations in $\mathbf{R}$.

$$|x-2|+3 < |x|+|x+1|$$

6.* Prove the following identities.

$$\sum_{k=1}^{n} k^2 = \tfrac{1}{6}n(n+1)(2n+1), \qquad \sum_{k=1}^{n} k^3 = (1+2+3+\cdots+n)^2.$$

7.* Prove that for every $a, b \in \mathbf{R}$ we have $|a+b| \leq |a|+|b|$ a $\big||a|-|b|\big| \leq |a-b|$.

8.* Let $x_1, \ldots, x_n \in \mathbf{R}$. Then we have $\left|\sum_{i=1}^{n} x_i\right| \leq \sum_{i=1}^{n} |x_i|$.

9.* Prove that for every $n \in \mathbf{N}$ we have $n \leq 2^n$.

10.* Prove that for every $n \in \mathbf{N}, n \neq 3$, we have $n^2 \leq 2^n$.

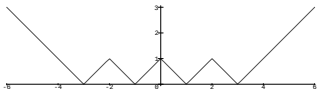
11.* Prove that $\sqrt{2}$ is irrational.

12.* (Binomial Theorem) For every $n \in \mathbf{N}$, $a, b \in \mathbf{R}$ we have

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k.$$

HINTS AND RESULTS

1. $(4,6\rangle$; $\langle 1,2\rangle$; $(-6,-3) \cup \big(\frac{1}{2}(-1-\sqrt{13}), \frac{1}{2}(-1+\sqrt{13})\big)$

2. Picture of the graph 

3. (1) $x = k\pi$ or $x = \frac{\pi}{3} + 2k\pi$ or $x = \frac{5\pi}{3} + 2k\pi$, where $k \in \mathbf{Z}$; (2) $x = 2k\pi$ or $x = \pi - \arcsin(4/5) + 2k\pi$, where $k \in \mathbf{Z}$; (3) $4/3$ **4.** $\cos 5x = \cos^5 x - 10\cos^3 x \sin^2 x + 5\cos x \sin^4 x$ a $\sin 5x = 5\cos^4 x \sin x - 10\cos^2 x \sin^3 x + \sin^5 x$. **5.** $(-\infty, -6) \cup (\frac{4}{3}, \infty)$ **6.** Use mathematical induction.