

Diferenční rovnice

Najděte všechna řešení diferenčních rovnic (s počátečními podmínkami).

- (1) $y(n+2) + 4y(n+1) + 4y(n) = 0$
- (2) $y(n+2) - 3y(n+1) + 2y(n) = 0$
- (3) $y(n+2) - 2y(n+1) - 3y(n) = 0, y(1) = 2, y(2) = 1$
- (4) $y(n+2) - y(n+1) - y(n) = 0, y(1) = y(2) = 1$
- (5) $y(n+4) + 6y(n+2) + 9y(n) = 0$
- (6) $y(n+2) - 2y(n+1) + 2y(n) = \cos \frac{\pi}{2}n$
- (7) $y(n+3) - y(n+2) - 2y(n+1) + 2y(n) = n + 2^n$
- (8) $8y(n+3) + y(n) = 3n + 1/2^n$
- (9) $y(n+2) - 3y(n+1) + 2y(n) = n^2, y(1) = 3, y(2) = 2.$
- (10) $y(n+2) - y(n) = 17, y(1) = y(2) = 0.$

Výsledky

1. $y(n) = c_1(-2)^n + c_2n(-2)^n, c_1, c_2 \in \mathbf{R}$
2. $y(n) = c_1 + c_22^n, c_1, c_2 \in \mathbf{R}$
3. $y(n) = -\frac{5}{4} \cdot (-1)^n + \frac{1}{4} \cdot 3^n,$
4. $y(n) = \frac{1}{\sqrt{5}} \left(\frac{1+\sqrt{5}}{2} \right)^n - \frac{1}{\sqrt{5}} \left(\frac{1-\sqrt{5}}{2} \right)^n$
5. $y(n) = c_1(\sqrt{3})^n \cos \frac{\pi}{2}n + c_2(\sqrt{3})^n \sin \frac{\pi}{2}n + c_3n(\sqrt{3})^n \cos \frac{\pi}{2}n + c_4n(\sqrt{3})^n \sin \frac{\pi}{2}n$
6. $y(n) = \frac{1}{5} \cos \frac{\pi}{2}n - \frac{2}{5} \sin \frac{\pi}{2}n + c_1(\sqrt{2})^n \cos \frac{\pi}{4}n + c_2(\sqrt{2})^n \sin \frac{\pi}{4}n$
7. $y(n) = -\frac{1}{2}n^2 - \frac{3}{2}n + 2^{n-1} + c_1(\sqrt{2})^n + c_2(-\sqrt{2})^n + c_3$
8. $y(n) = \frac{1}{3}n - \frac{8}{9} + \frac{1}{2}2^{-n} + c_1(-\frac{1}{2})^n + c_2(\frac{1}{2})^n \cos(\frac{\pi}{3}n) + c_3(\frac{1}{2})^n \sin(\frac{\pi}{3}n)$
9. $y(n) = -\frac{1}{3}n^3 - \frac{1}{2}n^2 - \frac{13}{6}n + 1 + 5 \cdot 2^{n-1}$
10. $y(n) = \frac{17}{2}n + \frac{51}{4} + \frac{85}{4}(-1)^n$