Chapter 1

Primitive functions

1.1 Solved problems

1. problem

- Compute $\int x^3 + 2x + \frac{17}{x} dx$.

Solution:

$$\int x^3 + 2x + \frac{17}{x} dx = \int x^3 dx + \int 2x dx + \int \frac{17}{x} dx$$
$$\stackrel{c}{=} \frac{1}{4}x^4 + x^2 + 17 \log |x|, \ x \in (-\infty, 0) \vee x \in (0, +\infty).$$

2. problem

- Compute $\int \sin^7 x \cos x dx$.

Solution:

$$\int \sin^7 x \cos x dx \stackrel{(1.1)}{=} \int y^7 dy \stackrel{c}{=} \frac{1}{8}y^8 = \frac{1}{8} \sin^8 x, \ x \in \mathbb{R}.$$
$$\sin x = y, \ \cos x dx = dy. \tag{1.1}$$

3. problem

- Compute $\int x e^x dx$.

Solution:

$$\int x e^x dx \stackrel{per\ partes}{=} x e^x - \int e^x dx \stackrel{c}{=} (x-1)e^x, \ x \in \mathbb{R}.$$

4. problem

- Compute $\int \frac{5e^{3x}+13e^{2x}+11e^x-2}{(e^x+1)(e^{2x}+2e^x+2)} dx$.

Solution:

$$\int \frac{5e^{3x} + 13e^{2x} + 11e^x - 2}{(e^x+1)(e^{2x}+2e^x+2)} dx \stackrel{(1.2)}{=} \int \frac{5y^3 + 13y^2 + 11y - 2}{(y+1)(y^2+2y+2)y} dy$$
$$= \int \frac{A}{y+1} + \frac{B}{y} + \frac{Cy+D}{y^2+2y+2} dy$$
$$\stackrel{(1.3)}{=} \int \frac{5}{y+1} dy - \int \frac{1}{y} dy + \int \frac{y+5}{y^2+2y+2} dy$$
$$\stackrel{c}{=} 5 \log |y+1| - \log |y| + \frac{1}{2} \log(y^2+2y+2) + 4 \arctan(y+1)$$

$$=5\log(e^x+1)-x+\frac{1}{2}\log(e^{2x}+2e^x+2)+4\arctan(e^x+1),\ x\in\mathbb{R}.$$

$$e^x=y,\ e^x dx=dy. \tag{1.2}$$

$$\begin{aligned}
A(y^2+2y+2)y+B(y^2+2y+2)(y+1)+&\\
(Cy+D)(y+1)y=&5y^3+13y^2+11y-2,\\
y:=0:\quad 2B=-2\implies& B=-1,\\
y:=-1:\quad -A=-5\implies& A=5,\\
y:=-1+i:\quad (C(-1+i)+D)i(-1+i)=&2C-D-iD=-3-5i\implies\\
D=5,\ C=1.&
\end{aligned} \tag{1.3}$$

$$\begin{aligned}
\int\frac{y+5}{y^2+2y+2}dy=&\frac{1}{2}\int 2y+2y^2+2y+2dy+4\int\frac{1}{(y+1)^2+1}dy\\
\overset{c}{=}&\frac{1}{2}\log(y^2+2y+2)+4\arctan(y+1).
\end{aligned}$$

5. problem

- Compute $\int\frac{e^x}{e^x+1}dx$.

Solution:

$$\int\frac{e^x}{e^x+1}dx\overset{(1.4)}{=}\int\frac{1}{y+1}dy\overset{c}{=}\log|y+1|=\log(e^x+1),\ x\in\mathbb{R}.$$

$$e^x=y,\ e^x dx=dy. \tag{1.4}$$

1.2 Unsolved problems

Compute following integrals:

(a) $\int xe^{-x^2}dx$,

(b) $\int 18e^x+16e^{8x}-\frac{1}{x}+3\cos x dx$,

(c) $\int\tan x dx$,

(d) $\int\frac{1}{\sin x}dx$,

(e) $\int\frac{x^{17}-5}{x-1}dx$,

(f) $\int\frac{x^3-1}{x^3-5x^2+6x}dx$,

(g) $\int\sqrt{\frac{1-x}{1+x}}dx$,

(h) $\int\frac{\cos^4 x+sin^4 x}{\cos^2 x-\sin^2 x}dx$,

(i) $\int\frac{dx}{1+\sqrt{x^2+2x+2}}$,

(j) $\int x^2\sin x dx$,

(k) $\int e^x\sin x dx$,

(l) $\int\frac{\sin^2 x+1}{\sin x(\cos x+1)}dx$,

(m) $\int\frac{\cos x+1}{\sin x-2}dx$,

(n) $\int\frac{7\log^4 x-2\log^3 x+32\log^2 x-14\log x+41}{x(\log^2 x+2\log x+5)(\log^2 x+3)(\log x-1)}dx$,

(o) $\int\frac{dx}{\cos x\sin^3 x}$ on interval $\left(0,\frac{\pi}{2}\right)$.