MULTIVARIATE CONTINUOUS DISTRIBUTIONS

1. Let consider the following bivariate continuous distribution which is defined for $1 \le y \le 2, 1 \le x \le 2$:

$$f_{X,Y}(x,y) = \frac{8x^2}{7y^3}$$

   (a) Check that it represent a density function
   (b) Compute the marginal $f_X(x)$ and check that it is a density function
   (c) Compute the marginal $f_Y(y)$ and check that it is a density function

2. Let consider the following bivariate continuous distribution which is defined for $0 \le y \le x \le 1$:

$$f_{X,Y}(x,y) = kxy$$

   (a) Find the value of k such that the given function represents a density function
   (b) Compute the marginal $f_X(x)$ and check that it is a density function
   (c) Compute the expected value of the marginal of $f_X(x)$
   (d) Compute the marginal $f_Y(y)$ and check that it is a density function
   (e) Compute the conditioned distribution $f(x|Y = 0.5)$ (defining correctly its support!)

3. Let consider the following bivariate continuous cumulative distribution which is defined for $0 \le y \le 1, 0 \le x \le 1$:

$$F_{X,Y}(x,y) = \frac{2}{3}y\left(\frac{1}{2}x^2 + x\right)$$

   (a) Find the corresponding bivariate continuous distribution $f_{X,Y}(x,y)$
   (b) Check that the bivariate continuous distribution $f_{X,Y}(x,y)$ that you just found is a density function
   (c) Compute the marginal $f_X(x)$ and check that it is a density function
   (d) Compute the marginal $f_Y(y)$ and check that it is a density function