The properties of probability

Geometric probability

1. We cut a one meter long pole randomly at two points. What is the probability that it is possible to construct a triangle from the resulting three pieces?

2. Two friends agreed to meet between 1pm and 2pm on a secret place. Each of them comes at a random time within this time period. Each of them waits 20 minutes for the other one and then leaves. What is the probability that the two friends meet?

Independence and conditional probability

4. Let's consider two events which are mutually exclusive. Are they independent?

5. The probability that in a train is not a place to sit is 0.2, and the probability that the same train arrives late is 0.3. The probability that the train arrives late or there is no place to sit is 0.4.

   (a) What is the probability train arrived on time, but you will not be able to sit in it?
   (b) How likely you will be able to sit on the train when arrived late?
   (c) Are the events "the train arrives on time"and "on the train will be place to seat"independent?

Probability space

6. Intersection of (countably many) $\sigma - algebra$ is again a $\sigma - algebra$. Union of $\sigma - algebras$ need not to be $\sigma - algebra$.

7. Show that classical probability space is probability space. Consider three dices. There are more possilbe classical probability spaces describing the random experiments.

8. Throw two dices and take sum of the results. Find classical probability space for the outcoming sum.

9. Throw two dices and take sum of the results. Find discrete probability space for the outcoming sum.

10. Show that $P(\cdot|B)$ is a probability space measure on Ω and $\mathcal{F}$.

11. Show that random events A and B are independent if

    (a) P(A) or P(B) is either 0 or 1.
    (b) P(A)=P($A|B$).

    Show that disjoint events cannot be independent unless (a) holds.

Review

Geometric probability:

- The state space Ω is identified as a geometric shape.
- The points lying inside correspond to elementary events having the same weight.
- The random events correspond to its subsets.
- The probability of event A is defined as the fraction of areas (volumes, …):

$$\mathsf{P}(A) = \frac{|A|}{|\Omega|}$$

Conditional probability:
Suppose A, B are random events with $\mathsf{P}(B) > 0$. The **conditional probability** of event A given B is defined as

$$\mathsf{P}(A|B) = \frac{\mathsf{P}(A \cap B)}{\mathsf{P}(B)}.$$

Law of total probability:
Let $A, B_1, B_2, \dots$ be random events such that $B_i \cap B_j = \emptyset$ for all $i \neq j$, $\bigcup_i B_i = \Omega$ and $\mathsf{P}(B_i) > 0$ for all $i = 1, 2, \dots$. Then

$$\mathsf{P}(A) = \sum_i \mathsf{P}(A \cap B_i) = \sum_i \mathsf{P}(A|B_i)\mathsf{P}(B_i).$$