CV@R Optimization problem

$$\min_{x} \quad -a + \frac{1}{\alpha}\sum_{s} z_s \cdot p_s$$ Maximization of the CV@R

$$-a + w_s + z_s \geq 0$$ CV@R definition

$$\frac{1}{S}\sum_{s} w_s \geq \widetilde{w}$$ Portfolio return target

$$w_s - w^0 \sum_{i} x_i \cdot \rho_{i,s} = 0$$ Portfolio return definition

$$z_s \geq 0$$ Slack variables must be positive

$$x_i \geq 0$$ Portfolio allocation must be positive

$$\sum_{i} x_i = 1$$ Portfolio weights sum to one

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$-a + \frac{1}{\alpha} \sum_s z_s \cdot p_s$$

$$-a + \frac{1}{\alpha} z_1 p_1 + \frac{1}{\alpha} z_2 p_2 + \cdots + \frac{1}{\alpha} z_S p_S$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$-a + \frac{1}{\alpha}\sum_s z_s \cdot p_s$$

$$-a + \frac{1}{\alpha} z_1 p_1 + \frac{1}{\alpha} z_2 p_2 + \cdots + \frac{1}{\alpha} z_S p_S$$

$$\underbrace{-1 + 0 + \cdots + 0}_{S} + \underbrace{\frac{1}{\alpha} p_1 + \frac{1}{\alpha} p_2 + \cdots + \frac{1}{\alpha} p_S}_{S} + \underbrace{0 + \cdots + 0}_{n}$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$-a + w_s + z_s \geq 0$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \ldots, w_S, z_1, z_2, \ldots, z_S, x_1, x_2, \ldots, x_n$

$$-a + w_s + z_s \geq 0$$

$$-a + w_1 + z_1 \geq 0 \qquad -1 + 1 + 0 + \cdots + 0 + 1 + 0 + \cdots + 0 + 0 + \cdots + 0 \geq 0$$

$$-a + w_2 + z_2 \geq 0 \qquad -1 + 0 + 1 + \cdots + 0 + 0 + 1 + \cdots + 0 + 0 + \cdots + 0 \geq 0$$

$$-a + w_S + z_S \geq 0 \qquad -1 + 0 + 0 + \cdots + 1 + 0 + 0 + \cdots + 1 + 0 + \cdots + 0 \geq 0$$

$$\underbrace{\qquad}_{S} \quad \underbrace{\qquad}_{S} \quad \underbrace{\qquad}_{n}$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$\frac{1}{S}\sum_s w_s \geq \widetilde{w}$$

$$\frac{1}{S}w_1 + \frac{1}{S}w_2 + \cdots + \frac{1}{S}w_S \geq \widetilde{w}$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$\frac{1}{S}\sum_s w_s \geq \widetilde{w}$$

$$\tfrac{1}{S}w_1 + \tfrac{1}{S}w_2 + \cdots + \tfrac{1}{S}w_S \geq \widetilde{w}$$

$$0 + \underbrace{\frac{1}{S} + \cdots + \frac{1}{S}}_{S} + \underbrace{0 + \cdots + 0}_{S} + \underbrace{0 + \cdots + 0}_{n} \geq \widetilde{w}$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$w_s - w^0 \sum_i x_i \cdot \rho_{i,s} = 0$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$w_s - w^0 \sum_i x_i \cdot \rho_{i,s} = 0$$

$$w_1 - x_1 w^0 \rho_{1,1} - x_2 w^0 \rho_{2,1} - \cdots - x_n w^0 \rho_{n,1} \geq 0 \qquad 0 + 1 + 0 + \cdots + 0 + 0 + 0 + \cdots + 0 - w^0 \rho_{1,1} - \cdots - w^0 \rho_{n,1} \geq 0$$

$$w_2 - x_1 w^0 \rho_{1,2} - x_2 w^0 \rho_{2,2} - \cdots - x_n w^0 \rho_{n,2} \geq 0 \qquad 0 + 0 + 1 + \cdots + 0 + 0 + 0 + \cdots + 0 - w^0 \rho_{1,2} - \cdots - w^0 \rho_{n,2} \geq 0$$

$$w_S - x_1 w^0 \rho_{1,S} - x_2 w^0 \rho_{2,S} - \cdots - x_n w^0 \rho_{n,S} \geq 0 \qquad \underbrace{0 + 0 + 0 + \cdots + 1}_{S} + \underbrace{0 + 0 + \cdots + 0}_{S} - \underbrace{w^0 \rho_{1,S} - \cdots - w^0 \rho_{n,S}}_{n} \geq 0$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$z_s \geq 0$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \ldots, w_S, z_1, z_2, \ldots, z_S, x_1, x_2, \ldots, x_n$

$$z_s \geq 0$$

$$z_1 \geq 0 \qquad 0 + 0 + \cdots + 0 + 1 + 0 + \cdots + 0 + 0 + \cdots + 0 \geq 0$$

$$z_2 \geq 0 \qquad 0 + 0 + \cdots + 0 + 0 + 1 + \cdots + 0 + 0 + \cdots + 0 \geq 0$$

$$z_S \geq 0 \qquad \underbrace{0 + 0 + \cdots + 0}_{S} + \underbrace{0 + 0 + \cdots + 1}_{S} + \underbrace{0 + \cdots + 0}_{n} \geq 0$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$x_i \geq 0$$

CV@R Optimization problem

Variables: $a, w_1, w_2, \ldots, w_S, z_1, z_2, \ldots, z_S, x_1, x_2, \ldots, x_n$

$$x_i \geq 0$$

$$x_1 \geq 0 \qquad 0 + 0 + \cdots + 0 + 0 + \cdots + 0 + 1 + 0 + \cdots + 0 \geq 0$$

$$x_2 \geq 0 \qquad 0 + 0 + \cdots + 0 + 0 + \cdots + 0 + 0 + 1 + \cdots + 0 \geq 0$$

$$x_n \geq 0 \qquad \underbrace{0 + 0 + \cdots + 0}_{S} + \underbrace{0 + \cdots + 0}_{S} + \underbrace{0 + 0 + \cdots + 1}_{n} \geq 0$$

Good luck!