

# CV@R Optimization problem

$$\min_{\mathbf{x}} \quad -a + \frac{1}{\alpha} \sum_s z_s \cdot p_s$$

Maximization of the CV@R

$$-a + w_s + z_s \geq 0$$

CV@R definition

$$\frac{1}{S} \sum_s w_s \geq \tilde{w}$$

Portfolio return target

$$w_s - w^0 \sum_i x_i \cdot \rho_{i,s} = 0$$

Portfolio return definition

$$z_s \geq 0$$

Slack variables must be positive

$$x_i \geq 0$$

Portfolio allocation must be positive

$$\sum_i x_i = 1$$

Portfolio weights sum to one

# CV@R Optimization problem

Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$-a + \frac{1}{\alpha} \sum_s z_s \cdot p_s$$

$$-a + \frac{1}{\alpha} z_1 p_1 + \frac{1}{\alpha} z_2 p_2 + \dots + \frac{1}{\alpha} z_S p_S$$

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Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$-a + \frac{1}{\alpha} \sum_s z_s \cdot p_s$$

$$-a + \frac{1}{\alpha} z_1 p_1 + \frac{1}{\alpha} z_2 p_2 + \dots + \frac{1}{\alpha} z_S p_S$$

$$-1 + 0 + \dots + 0 + \frac{1}{\alpha} p_1 + \frac{1}{\alpha} p_2 + \dots + \frac{1}{\alpha} p_S + 0 + \dots + 0$$


$S$              $S$              $n$

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Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$-a + w_s + z_s \geq 0$$

# CV@R Optimization problem

Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$-a + w_s + z_s \geq 0$$

$$-a + w_1 + z_1 \geq 0$$

$$-1 + 1 + 0 + \cdots + 0 + 1 + 0 + \cdots + 0 + 0 + \cdots + 0 \geq 0$$

$$-a + w_2 + z_2 \geq 0$$

$$-1 + 0 + 1 + \cdots + 0 + 0 + 1 + \cdots + 0 + 0 + \cdots + 0 \geq 0$$

$$-a + w_S + z_S \geq 0$$

$$-1 + 0 + 0 + \cdots + 1 + 0 + 0 + \cdots + 1 + 0 + \cdots + 0 \geq 0$$



$S$



$S$



$n$

# CV@R Optimization problem

Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$\frac{1}{S} \sum_s w_s \geq \tilde{w}$$

$$\frac{1}{S} w_1 + \frac{1}{S} w_2 + \dots + \frac{1}{S} w_S \geq \tilde{w}$$

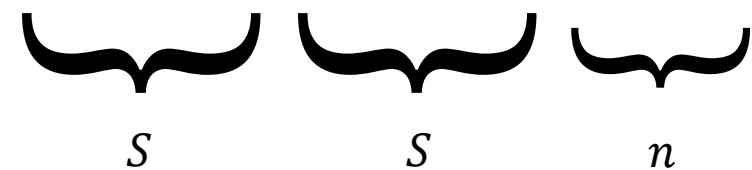
# CV@R Optimization problem

Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$\frac{1}{S} \sum_s w_s \geq \tilde{w}$$

$$\frac{1}{S} w_1 + \frac{1}{S} w_2 + \dots + \frac{1}{S} w_S \geq \tilde{w}$$

$$0 + \frac{1}{S} + \dots + \frac{1}{S} + 0 + \dots + 0 + 0 + \dots + 0 \geq \tilde{w}$$



# CV@R Optimization problem

Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$w_s - w^0 \sum_i x_i \cdot \rho_{i,s} = 0$$

# CV@R Optimization problem

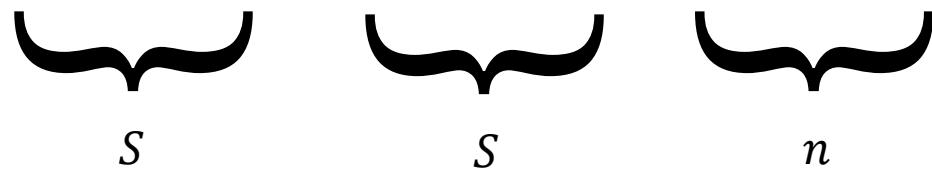
Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$w_s - w^0 \sum_i x_i \cdot \rho_{i,s} = 0$$

$$w_1 - x_1 w^0 \rho_{1,1} - x_2 w^0 \rho_{2,1} - \dots - x_n w^0 \rho_{n,1} \geq 0 \quad 0 + 1 + 0 + \dots + 0 + 0 + 0 + \dots + 0 - w^0 \rho_{1,1} - \dots - w^0 \rho_{n,1} \geq 0$$

$$w_2 - x_1 w^0 \rho_{1,2} - x_2 w^0 \rho_{2,2} - \dots - x_n w^0 \rho_{n,2} \geq 0 \quad 0 + 0 + 1 + \dots + 0 + 0 + 0 + \dots + 0 - w^0 \rho_{1,2} - \dots - w^0 \rho_{n,2} \geq 0$$

$$w_S - x_1 w^0 \rho_{1,S} - x_2 w^0 \rho_{2,S} - \dots - x_n w^0 \rho_{n,S} \geq 0 \quad 0 + 0 + 0 + \dots + 1 + 0 + 0 + \dots + 0 - w^0 \rho_{1,S} - \dots - w^0 \rho_{n,S} \geq 0$$



# CV@R Optimization problem

Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$z_s \geq 0$$

# CV@R Optimization problem

Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$z_s \geq 0$$

$$z_1 \geq 0$$

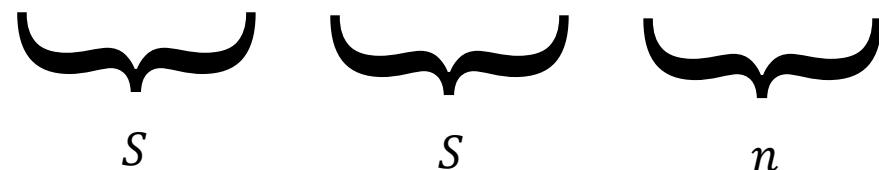
$$0 + 0 + \cdots + 0 + 1 + 0 + \cdots + 0 + 0 + \cdots + 0 \geq 0$$

$$z_2 \geq 0$$

$$0 + 0 + \cdots + 0 + 0 + 1 + \cdots + 0 + 0 + \cdots + 0 \geq 0$$

$$z_S \geq 0$$

$$0 + 0 + \cdots + 0 + 0 + 0 + \cdots + 1 + 0 + \cdots + 0 \geq 0$$



# CV@R Optimization problem

Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$x_i \geq 0$$

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Variables:  $a, w_1, w_2, \dots, w_S, z_1, z_2, \dots, z_S, x_1, x_2, \dots, x_n$

$$x_i \geq 0$$

$$x_1 \geq 0$$

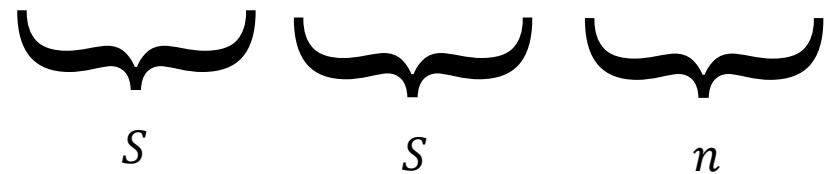
$$0 + 0 + \cdots + 0 + 0 + \cdots + 0 + 1 + 0 + \cdots + 0 \geq 0$$

$$x_2 \geq 0$$

$$0 + 0 + \cdots + 0 + 0 + \cdots + 0 + 0 + 1 + \cdots + 0 \geq 0$$

$$x_n \geq 0$$

$$0 + 0 + \cdots + 0 + 0 + \cdots + 0 + 0 + 0 + \cdots + 1 \geq 0$$



Good luck!