Stochastic Analysis and Its Applications XVI
Room Praktikum (First Floor)
KPMS MFF UK, Sokolovska 83, Praha 8
January 3, 2020
Program

9:30 – 10:00: Arrival, Coffee

10:00 – 10:40
Jan Swart, Balázs Ráth, Tamás Terpai
Frozen percolation on the 3-regular tree

10:40 – 11:10
Petr Dostál
On Convergence of Langevin Algorithm

11:10 – 11:50
Ondřej Týbl
Kalman-Bucy Filter in Continuous Time

11:50 – 13:00: Lunch Break

13:00 – 13:40:
Dalibor Volný
Martingale approximation and limit theorems

13:40 – 14:20:
Jan Večeř, Robert Navrátil
Utility Based Model Selection and Model Averaging

14:20 – 15:00:
Jan Pospíšil
Solution of option pricing equations using orthogonal polynomial expansion

15:00 – 15:10: Coffee Break

15:10 – 15:50:
Jiří Hozman, **Tomáš Tichý**
DG method for pricing options in jump-diffusion models

15:50 – 16:30:
Viktor Beneš, Christoph Hofer-Temmel, Guenter Last, Jakub Veceřa
Decorrelation of a class of Gibbs particle processes and asymptotic properties of U-statistics

16:30 – 16:40: Coffee Break

16:40 – 17:20:
Martin Šmíd
Local Interactions of Pure-Jump Processes - Application to Batch Auction

17:20 – 18:00:
Robert Navrátil, Jan Večeř
Long term portfolio protection

18:00: Dinner

Abstracts

Jan Swart, Balázs Ráth, Tamás Terpai

Frozen percolation on the 3-regular tree

In frozen percolation, i.i.d. uniformly distributed activation times are assigned to the edges of a graph. At its assigned time, an edge opens provided neither of its endvertices is part of an infinite open cluster; in the opposite case, it freezes. David Aldous (2000) showed that such a process can be constructed on the infinite 3-regular tree and asked whether the event that a given edge freezes is a measurable function of the activation times assigned to all edges. We give a negative answer to this question.

Petr Dostál

On Convergence of Langevin Algorithm

The talk is devoted to the pure (unadjusted) Langevin algorithm. We show how it can be used in order to approximate the target distribution under the assumption that the corresponding drift coefficient has bounded and Lipschitz continuous coordinates. We offer a transform of variables that significantly weakens this assumption so that the algorithm can be applied (after a transform) to the cases when the logarithm of the smooth target density has at most polynomial growth as well as its derivatives up to the second order. Finally, we offer transforms of variables in order to ensure that the algorithm can be applied to further examples.

Ondřej Týbl

Kalman-Bucy Filter in Continuous Time

The aim of the work is to study a linear filtering problem for Gaussian processes in finite-dimensional spaces in two aspects: continuous dependence of the filter on the signal and existence and uniqueness of a solution to an integral equation that defines the covariance of the error and the signal. We extend the results from Kubelka and Maslowski [1] in some directions. The covariance of observation error as derived in [1] satisfies non-linear integral equation with non-Lipschitz right-hand side, which does not allow us to use the classical method of Gronwall inequality to derive continuous dependence of the solution on a parameter. In such a complex situation we utilize the notion of relative compactness and a stochastic integral representation of the filter. This result is then extended to prove the continuous dependence of the filter itself. Moreover, existence and uniqueness of the solution of the integral equation inherited from [1] under more general assumptions is shown using the Banach Fixed point Theorem. This presented approach provides directly a numerical method for finding the solution with exponential speed of convergence.

[1] Maslowski B., Kubelka V.: Filtering of gaussian processes in Hilbert spaces. arXiv preprint:1903.11464, 2019

Dalibor Volný

Martingale approximation and limit theorems

We study central limit theorems and (weak) invariance principles for stationary sequences of random variables and for random fields. In the case of random fields there are several ways of defining a filtration; in the talk we consider completely commuting filtration as defined by Khosnevisan.

Jan Večeř, Robert Navrátil

Utility Based Model Selection and Model Averaging

This talk presents a novel approach to model selection and model averaging based on economic theory. We study model prediction in the form of a distributional opinion about a random variable X. We show how to test this prediction against alternative views. Different model opinions can be traded on a hypothetical market that trades their differences. Using a utility maximization technique, we describe such a market for any general random variable X and any utility function U. We specify the optimal behavior of agents and the total market that aggregates all available opinions and show that a correct distributional opinion realizes profit in expectation against any other opinion, giving a novel technique for model selection. The expected profit from this trading defines statistical divergence. In particular, exponential utility gives divergence that is very close to Kullback-Leibler and the logarithmic utility gives a novel f-divergence. Analytical solutions are available for random variables from the exponential family. We also determine the distribution corresponding to the aggregated view of all available opinions, giving a novel technique for model averaging.

Jan Pospíšil

Solution of option pricing equations using orthogonal polynomial expansion

In this talk we show both analytic and numerical solutions of option pricing equations using systems of orthogonal polynomials. Using a Galerkin-based method, we solve the parabolic partial diferential equation for the Black-Scholes model using Hermite polynomials and for the Heston model using Hermite and Laguerre polynomials. We compare obtained solutions to existing semi-closed pricing formulas. Special attention is paid to the solution of Heston model at the boundary with vanishing volatility.

Jiří Hozman, **Tomáš Tichý**

DG method for pricing options in jump-diffusion models

Jump-diffusion (JD) models extend the standard Black-Scholes (BS) framework by adding jumps to the dynamics of underlying asset prices and enable to describe large and sudden changes in the underlying. The valuation of options under such a model requires solving a parabolic partial integro-differential equation (PIDE) which involves both the integrals and the derivatives of the unknown pricing function.

This presentation is devoted to the discontinuous Galerkin (DG) method applied to European option pricing under two specific jump-diffusion models, namely the Merton JD model and the Kou JD model. After a localization of the pricing PIDE to a bounded spatial domain, the governing equation is discretized by the DG method over a finite element mesh. However, the integral term related to jumps leads to new theoretical and numerical issues regarding the solving of the pricing equation in comparison with the standard approach for the BS equation.

Here we present two different approaches how to approximate the integral term. In the case of the Kou model, the semidiscrete problem is integrated in temporal variable by a semi-implicit Euler scheme, where the differential part is treated implicitly while the integral one explicitly by the composite trapezoidal rule. On the other hand for the Merton model, we adopt the idea that the integral terms in Merton-type models can be viewed as solutions of proper differential equations, which can be accurately solved in a simple way. Then the semidiscrete problem is coupled with a two-stage implicit-explicit time stepping scheme.

The entire solution procedure (for both schemes presented) is accompanied with theoretical results and discussed within the numerical results on reference benchmarks.

Viktor Beneš, Christoph Hofer-Temmel, Guenter Last, Jakub Veceřa

Decorrelation of a class of Gibbs particle processes and asymptotic properties of U-statistics

We investigate a stationary Gibbs particle process with deterministically bounded particles on Euclidean space defined in terms of non-negative interaction potentials and an activity parameter. For small activity parameters, we study the asymptotics for certain U-statistics of this Gibbs particle process with increasing window of observation. To this end we establish an exponential decorrelation property, a result of independent interest.

Martin Šmíd

Local Interactions of Pure-Jump Processes - Application to Batch Auction

Modelling today's financial market brings a new theoretical challenge: rigorous description of simultaneous non-Markov pure jump processes, taking values in more-than-Euclidean spaces, asymmetrically influencing each other. In the present paper, we develop a theory of such processes, give a formula for the conditional distribution of a process, locally influenced by another one (not influencing in return), and formulate sufficient conditions for independence of two processes running alongside. Further, we discuss jump processes of real random measures and give sufficient conditions for complete independence of snapshots of these processes. Finally, we formulate a model of the batch auction, in which order-books are ruled by jump processes of random measures, directed by another process (say fair price). We formulate a semi-analytic formula for the distribution of the fair price and demonstrate the model numerically.

Robert Navrátil, Jan Večeř

Long Term Portfolio Protection

The purpose of this talk is to discuss novel approaches how to protect potential portfolio losses on long term horizons in the scale of several decades, which is a typical investment horizon of pension fund investments. The prices of existing financial products, such as put or call options, are increasing as a function of maturity, and their prices quickly take a significant percentage of the underlying assets. In this respect, such financial products become prohibitively expensive on horizons longer than a couple of years at most. In addition, these contracts tend to insure only static rather than actively traded portfolios which are more appropriate for pension funds. Thus it is desirable to have a protection of actively traded portfolio, where the client is free to move her wealth within different asset classes, while the portfolio value is protected against any trading losses. This is a generalization of a previously studied contract known as a passport option, but in our setup, the price of this contract is small enough to be attractive on 20-30 year investment horizons and thus the respective hedging strategy can be potentially embedded in pension fund products. motion.