CVIČENÍ Z MATEMATICKÉ ANALÝZY 2

PRIMITIVNÍ FUNKCE

Nalezněte následující primitivní funkce na maximálních intervalech, kde existují.

(1) $\int (x^9 + \frac{1}{x} - 5e^x + x^{-3} - \cos x)\,\mathrm{d}x$

(2) $\int x^2 \cos x\,\mathrm{d}x$

(3) $\int \frac{x}{1+x^4}\,\mathrm{d}x$

(4) $\int |x|\,\mathrm{d}x$

(5) $\int \frac{x^2+3x+6}{x^4}\,\mathrm{d}x$

(6) $\int \frac{1}{\sqrt{2-5x}}\,\mathrm{d}x$

(7) $\int \frac{1}{2x+3}\,\mathrm{d}x$

(8) $\int x(1-x)^{10}\,\mathrm{d}x$

(9) $\int \frac{1}{x^2+2x+2}\,\mathrm{d}x$

(10) $\int \frac{x^2}{1+x^2}\,\mathrm{d}x$

(11) $\int (2e^{3x} - \sqrt[5]{5-x})\,\mathrm{d}x$

(12) $\int \max\{x, x^2\}\,\mathrm{d}x$

(13) $\int |\cos x|\,\mathrm{d}x$

(14) $\int |2x+1|\,\mathrm{d}x$

(15) $\int x \operatorname{arctg} x\,\mathrm{d}x$

(16) $\int x^3 \sin x\,\mathrm{d}x$

(17) $\int e^x \cos x\,\mathrm{d}x$

(18) $\int \log x\,\mathrm{d}x$

(19) $\int x^n e^x\,\mathrm{d}x$, $n \in \mathbb{N}$

(20) $\int \operatorname{arctg} x\,\mathrm{d}x$

(21) $\int \cos^2 x\,\mathrm{d}x$

(22) $\int \frac{\operatorname{arctg} x}{1+x^2}\,\mathrm{d}x$

(23) $\int x^2 \sin(2x)\,\mathrm{d}x$

(24) $\int \cos^5(x)\sqrt{\sin x}\,\mathrm{d}x$

(25) $\int e^{ax} \cos(bx)\,\mathrm{d}x$, $a,b \in \mathbb{R}$

(26) $\int \frac{\operatorname{arctg} e^x}{e^x}\,\mathrm{d}x$

(27) $\int \operatorname{tg} x\,\mathrm{d}x$

(28) $\int \frac{1}{x \log x}\,\mathrm{d}x$

(29) $\int \frac{x}{\sqrt{x^2+5}}\,\mathrm{d}x$

(30) $\int (2^x + 3^x)^2\,\mathrm{d}x$

(31) $\int \frac{\log x}{x\sqrt{1+\log x}}\,\mathrm{d}x$

(32) $\int \frac{1}{\sin x}\,\mathrm{d}x$

(33) $\int \frac{\sin x}{1+\sin^4 x} \cos x\,\mathrm{d}x$

(34) $\int \frac{2x}{2+2x^2+x^4}\,\mathrm{d}x$

(35) $\int \frac{\sin x}{\sqrt{\cos^3 x}}\,\mathrm{d}x$

(36) $\int \frac{x^2}{(8x^3+27)^{2/3}}\,\mathrm{d}x$

(37) $\int x^5 e^{x^3}\,\mathrm{d}x$

(38) $\int e^{\sqrt{x}}\,\mathrm{d}x$

(39) $\int \frac{1}{x^2} \sin \frac{1}{x}\,\mathrm{d}x$

(40) $\int \frac{x^3}{x^8+1}\,\mathrm{d}x$

(41) $\int \sqrt{4-x^2}\,\mathrm{d}x$

(42) $\int \sqrt{x^2+a^2}\,\mathrm{d}x$

(43) $\int \frac{x^3-4x-6}{x^3-5x^2+6x}\,\mathrm{d}x$

(44) $\int \frac{5x^3+3x^2-x-1}{x^2+2x+1}\,\mathrm{d}x$

(45) $\int \frac{1}{x^4-1}\,\mathrm{d}x$

(46) $\int \frac{1}{x^4+1}\,\mathrm{d}x$

(47) $\int \frac{x^2+x}{x^6+3x^4+3x^2+1}\,\mathrm{d}x$

(48) $\int \frac{e^x}{1+e^{2x}}\,\mathrm{d}x$

(49) $\int \frac{1}{e^{2x}+e^x-2}\,\mathrm{d}x$

(50) $\int \frac{1}{1+e^{x/2}+e^{x/3}+e^{x/6}}\,\mathrm{d}x$

(51) $\int \frac{1}{\cos x \sin^2 x}\,\mathrm{d}x$

(52) $\int \frac{\sin^3 x+\sin x}{\cos^3 x+\cos x}\,\mathrm{d}x$

(53) $\int \frac{1}{x(\log^2 x-1)}\,\mathrm{d}x$

(54) $\int \frac{\sin^2 x}{1+\sin^2 x}\,\mathrm{d}x$

(55) $\int \frac{1}{2\sin x-\cos x+5}\,\mathrm{d}x$

(56) $\int \frac{1}{\sin x+\operatorname{tg} x}\,\mathrm{d}x$

(57) $\int \frac{1}{1+\sqrt{x}}\,\mathrm{d}x$

(58) $\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$

(59) $\int \sqrt{\frac{1-e^{2x}}{e^{2x}+2e^x+1}}\,\mathrm{d}x$

(60) $\int \frac{1}{1+\sqrt{x^2+2x+2}}\,\mathrm{d}x$

(61) $\int \frac{1}{(x-1)\sqrt{x^2-3x+2}}\,\mathrm{d}x$

<center>VÝSLEDKY</center>

Výsledky jsou uvedeny „až na konstantu“, tzn. $F(x)$ znamená, že primitivní funkce jsou právě všechny funkce tvaru $F(x) + C$, kde $C \in \mathbb{R}$.

(1) $\frac{x^{10}}{10} + \log|x| - 5e^x - \frac{1}{2x^2} - \sin x$ na $(-\infty, 0)$ a $(0, \infty)$

(2) $x^2 \sin x + 2x \cos x - 2 \sin x$ na $\mathbb{R}$

(3) $\frac{1}{2} \operatorname{arctg} x^2$, $x \in \mathbb{R}$

(4) $\operatorname{sign} x \cdot \frac{x^2}{2}$ na $\mathbb{R}$

(5) $-\frac{1}{x} - \frac{3}{2x^2} - \frac{2}{x^3}$ na $(-\infty, 0)$ a $(0, \infty)$

(6) $-\frac{2}{5}\sqrt{2 - 5x}$ na $(-\infty, \frac{2}{5})$

(7) $\frac{1}{2} \log|2x + 3|$ na $\left(-\infty, -\frac{3}{2}\right)$ a $\left(-\frac{3}{2}, \infty\right)$

(8) $-\frac{(1-x)^{11}}{11} + \frac{(1-x)^{12}}{12}$ na $\mathbb{R}$

(9) $\operatorname{arctg}(x + 1)$ na $\mathbb{R}$

(10) $x - \operatorname{arctg} x$ na $\mathbb{R}$

(11) $\frac{2}{3}e^{3x} + \frac{5(5-x)^{\frac{6}{5}}}{6}$ na $\mathbb{R}$

(12) $F(x) = \begin{cases} \frac{x^3}{3} & x \in (-\infty, 0) \\ \frac{x^2}{2} & x \in [0, 1] \\ \frac{x^3}{3} + \frac{1}{6} & x \in (1, \infty) \end{cases}$

(13) $F(x) = \begin{cases} \sin x + 4k & x \in [-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi], k \in \mathbb{Z} \\ -\sin x + 4k + 2 & x \in (\frac{\pi}{2} + 2k\pi, 3\frac{\pi}{2} + 2k\pi), k \in \mathbb{Z} \end{cases}$

(14) $F(x) = \begin{cases} -(x^2 + x) & x < -\frac{1}{2} \\ x^2 + x + \frac{1}{2} & x \geq -\frac{1}{2} \end{cases}$

(15) $\frac{x^2}{2} \operatorname{arctg} x - \frac{x}{2} + \frac{\operatorname{arctg} x}{2}$ na $\mathbb{R}$

(16) $-x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x$ na$\mathbb{R}$

(17) $\frac{1}{2}(e^x \sin x + e^x \cos x)$ na $\mathbb{R}$

(18) $x \log x - x$ na $(0, \infty)$

(19) $I_n := \int x^n e^x \,\mathrm{d}x = x^n e^x - nI_{n-1}$; $I_1 := xe^x - e^x$ na $\mathbb{R}$

(20) $x \operatorname{arctg} x - \frac{1}{2} \log(1 + x^2)$ na $\mathbb{R}$

(21) $\frac{x}{2} + \frac{\sin(2x)}{4}$ na $\mathbb{R}$

(22) $\frac{1}{2} \operatorname{arctg}^2 x$ na $\mathbb{R}$

(23) $-\frac{2x^2-1}{4} \cos(2x) + \frac{x}{2} \sin(2x)$ na $\mathbb{R}$

(24) $\frac{2}{3}(\sin x)^{\frac{3}{2}} - \frac{4}{7}(\sin x)^{\frac{7}{2}} + \frac{2}{11}(\sin x)^{\frac{11}{2}}$ na $(2k\pi, (2k+1)\pi)$, $k \in \mathbb{N}$

(25) $\frac{e^{ax}}{a^2+b^2}(a \cos(bx) + b \sin(ax))$ na $\mathbb{R}$

(26) $-e^{-x} \operatorname{arctg} e^x - \frac{1}{2} \log \frac{e^{2x}}{1+e^{2x}}$ na $\mathbb{R}$

(27) $-\log|\cos x|$ na $\left(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi\right)$, $k \in \mathbb{Z}$

(28) $\log|\log x|$ na $(0, 1)$ a $(1, \infty)$

(29) $\sqrt{x^2 + 5}$ na $\mathbb{R}$

(30) $\frac{4^x}{\log 4} + 2\frac{6^x}{\log 6} + \frac{9^x}{\log 9}$ na $\mathbb{R}$

(31) $\frac{2}{3}(1 + \log x)^{\frac{3}{2}} - 2(1 + \log x)^{\frac{1}{2}}$ na $\left(\frac{1}{e}, \infty\right)$

(32) $\log\left|\operatorname{tg} \frac{x}{2}\right|$ na $(k\pi, (k+1)\pi)$, $k \in \mathbb{N}$

(33) $\frac{1}{2} \operatorname{arctg}(\sin^2 x)$ na $\mathbb{R}$

(34) $\operatorname{arctg}(x^2 + 1)$ na $\mathbb{R}$

(35) $\frac{2}{\sqrt{\cos x}}$ na $\left(-\frac{\pi}{2} + 2k\pi, \frac{\pi}{2} + 2k\pi\right)$, $k \in \mathbb{Z}$

(36) $\frac{1}{8}\sqrt[3]{8x^3 + 27}$ na $(-\infty, -\frac{3}{2})$, $(-\frac{3}{2}, \infty)$

(37) $\frac{1}{3}(x^3 - 1)e^{x^3}$ na $\mathbb{R}$

(38) $2(\sqrt{x} - 1)e^{\sqrt{x}}$ na $(0, \infty)$

(39) $\cos \frac{1}{x}$ na $(-\infty, 0)$, $(0, \infty)$

(40) $\frac{1}{4} \operatorname{arctg}(x^4)$ na $\mathbb{R}$

(41) $2 \arcsin \frac{x}{2} + \sin(2 \arcsin \frac{x}{2}) = 2 \arcsin \frac{x}{2} + \frac{x}{2}\sqrt{4 - x^2}$ na $(-2, 2)$

(42) $\frac{x}{2}\sqrt{x^2+a^2}+\frac{a^2}{2}\log(x+\sqrt{x^2+a^2})$ na $\mathbb{R}$

(43) $x-\log|x|+3\log|x-3|+3\log|x-2|$ na $(-\infty,0),\,(0,2),\,(2,3),\,(3,\infty)$

(44) $5\frac{x^2}{2}-7x+8\log|x+1|+2\frac{1}{x+1}$ na $(-\infty,-1),\,(-1,\infty)$

(45) $\frac{1}{4}\log|x-1|-\frac{1}{4}\log|x+1|-\frac{1}{2}\operatorname{arctg} x$ na $(-\infty,-1),\,(-1,1),\,(1,\infty)$

(46) $\frac{\sqrt{2}}{8}\log(x^2+\sqrt{2}x+1)+\frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}x+1)-\frac{\sqrt{2}}{8}\log(x^2-\sqrt{2}x+1)+\frac{1}{2\sqrt{2}}\operatorname{arctg}(\sqrt{2}x-1)$ na $\mathbb{R}$

(47) $I_2-I_3-\frac{1}{4(1+x^2)^2}$, kde $I_n=\int\frac{1}{(1+x^2)^n}\,\mathrm{d}x$

(48) $\operatorname{arctg} e^x$, $x\in\mathbb{R}$

(49) $-\frac{x}{2}+\frac{1}{3}\log|e^x-1|+\frac{1}{6}\log(e^x+2)$ na $(-\infty,0),\,(0,\infty)$

(50) $x-3\log(e^{x/6}+1)-3\log(\sqrt{e^{x/3}+1})-3\operatorname{arctg}(e^{x/6})$ na $\mathbb{R}$

(51) $-\frac{1}{\sin x}-\frac{1}{2}\log|1-\sin x|+\frac{1}{2}\log|1+\sin x|$ na $\left(k\frac{\pi}{2},(k+1)\frac{\pi}{2}\right)$, $k\in\mathbb{Z}$

(52) $\frac{3}{2}\log(\cos^2 x+1)-\log(\cos^2 x)$ na $\left(k\pi-\frac{\pi}{2},k\pi+\frac{\pi}{2}\right)$, $k\in\mathbb{Z}$

(53) $\frac{1}{2}\log|\log x-1|-\frac{1}{2}\log|\log x+1|$ na $\left(0,\frac{1}{e}\right),\,\left(\frac{1}{e},e\right),\,(e,\infty)$

(54) $\begin{cases} \operatorname{arctg}(\operatorname{tg} x)-\frac{1}{\sqrt{2}}\operatorname{arctg}(\sqrt{2}\operatorname{tg} x)+k\pi(1-1/\sqrt{2}) & x\in\left(-\frac{\pi}{2}+k\pi,\frac{\pi}{2}+k\pi\right), \quad k\in\mathbb{Z}\\ \frac{\pi}{2}-\frac{\pi}{2\sqrt{2}}+k\pi(1-1/\sqrt{2}) & x=\frac{\pi}{2}+k\pi, \quad k\in\mathbb{Z} \end{cases}$

(55) $F(x)=\begin{cases} \frac{\sqrt{5}}{5}\operatorname{arctg}\left(\frac{3\operatorname{tg}\frac{x}{2}+1}{\sqrt{5}}\right)+k\pi\frac{\sqrt{5}}{5} & \text{pro } x\in(-\pi+2k\pi,\pi+2k\pi),\\ \frac{\pi}{2}\frac{\sqrt{5}}{5}+k\pi\frac{\sqrt{5}}{5} & \text{pro } x=\pi+2k\pi, k\in\mathbb{Z} \end{cases}$

(56) $\frac{1}{4}\log\left|\frac{1-\cos x}{1+\cos x}\right|-\frac{1}{2(\cos x+1)}$ na $\left(k\pi-\frac{\pi}{2},k\pi+\frac{\pi}{2}\right)$, $k\in\mathbb{Z}$

(57) $2\sqrt{x}-2\log(1+\sqrt{x})$ na $(0,\infty)$

(58) $-2\operatorname{arctg}\sqrt{\frac{1-x}{1+x}}-\log\left|1-\sqrt{\frac{1-x}{1+x}}\right|+\log\left|1+\sqrt{\frac{1-x}{1+x}}\right|$ na $(-1,0),\,(0,1)$

(59) $-2\operatorname{arctg}\sqrt{\frac{1-e^x}{1+e^x}}-\log\left|1-\sqrt{\frac{1-e^x}{1+e^x}}\right|+\log\left|1+\sqrt{\frac{1-e^x}{1+e^x}}\right|$ na $(-\infty,0)$

(60) $\frac{2}{x-\sqrt{x^2+2x+2}}-\log(\sqrt{x^2+2x+2}-x-1)$ na $\mathbb{R}$

(61) $2\operatorname{sign}(x-1)\sqrt{\frac{x-2}{x+1}}$ na $(-\infty,1),\,(2,\infty)$