Vypočtěte parciální derivace prvního a druhého řádu následujících funkcí:

3213. $u = x^4 + y^4 - 4x^2y^2$.

3214. $u = xy + \frac{x}{y}$.

3215. $u = \frac{x}{y^2}$.

3216. $u = \frac{x}{\sqrt{x^2+y^2}}$.

3217. $u = x\sin(x+y)$.

3218. $u = \frac{\cos x^2}{y}$.

3219. $u = \operatorname{tg}\frac{x^2}{y}$.

3220. $u = x^y$.

3221. $u = \ln(x+y^2)$.

3222. $u = \operatorname{arctg}\frac{y}{x}$.

3223. $u = \operatorname{arctg}\frac{x+y}{1-xy}$.

3224. $u = \arcsin\frac{x}{\sqrt{x^2+y^2}}$.

3225. $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$.

3226. $u = \left(\frac{x}{y}\right)^2$.

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není diferencovatelná v bodě $(0,0)$. **3212.2** Funkce není diferencovatelná v bodě $(0,0)$. **3212.3** Funkce je diferencovatelná v bodě $(0,0)$. **3213.** $\frac{\delta u}{\delta x} = 4x^3 - 8xy^2$, $\frac{\delta u}{\delta y} = 4y^3 - 8x^2y$, $\frac{\delta^2 u}{\delta x^2} = 12x^2 - 8y^2$, $\frac{\delta^2 u}{\delta x \delta y} = -16xy$, $\frac{\delta^2 u}{\delta y^2} = 12y^2 - 8x^2$. **3214.** $\frac{\delta u}{\delta x} = y + \frac{1}{y}$, $\frac{\delta u}{\delta y} = x - \frac{x}{y^2}$, $\frac{\delta^2 u}{\delta x^2} = 0$, $\frac{\delta^2 u}{\delta x \delta y} = 1 - \frac{1}{y^2}$, $\frac{\delta^2 u}{\delta y^2} = \frac{2x}{y^3}$. **3215.** $\frac{\delta u}{\delta x} = \frac{1}{y^2}$, $\frac{\delta u}{\delta y} = -\frac{2x}{y^3}$, $\frac{\delta^2 u}{\delta x^2} = 0$, $\frac{\delta^2 u}{\delta x \delta y} = -\frac{2}{y^3}$, $\frac{\delta^2 u}{\delta y^2} = \frac{6x}{y^4}$. **3216.** $\frac{\delta u}{\delta x} = \frac{y^2}{(x^2+y^2)^{3/2}}$, $\frac{\delta u}{\delta y} = -\frac{xy}{(x^2+y^2)^{3/2}}$, $\frac{\delta^2 u}{\delta x^2} = -\frac{3xy^2}{(x^2+y^2)^{5/2}}$, $\frac{\delta^2 u}{\delta x \delta y} = \frac{y(2x^2-y^2)}{(x^2+y^2)^{5/2}}$, $\frac{\delta^2 u}{\delta y^2} = -\frac{x(x^2-2y^2)}{(x^2+y^2)^{5/2}}$. **3217.** $\frac{\delta u}{\delta x} = \sin(x+y) + x\cos(x+y)$, $\frac{\delta u}{\delta y} = x\cos(x+y)$, $\frac{\delta^2 u}{\delta x^2} = 2\cos(x+y) - x\sin(x+y)$, $\frac{\delta^2 u}{\delta x \delta y} = \cos(x+y) - x\sin(x+y)$, $\frac{\delta^2 u}{\delta y^2} = -x\sin(x+y)$. **3218.** $\frac{\delta u}{\delta x} = -\frac{2x\sin x^2}{y}$, $\frac{\delta u}{\delta y} = -\frac{\cos x^2}{y^2}$, $\frac{\delta^2 u}{\delta x^2} = -\frac{2\sin x^2 + 4x^2\cos x^2}{y}$, $\frac{\delta^2 u}{\delta x \delta y} = \frac{2x\sin x^2}{y^2}$, $\frac{\delta^2 u}{\delta y^2} = \frac{2\cos x^2}{y^3}$. **3219.** $\frac{\delta u}{\delta x} = \frac{2x}{y}\sec^2\frac{x^2}{y}$, $\frac{\delta u}{\delta y} = -\frac{x^2}{y^2}\sec^2\frac{x^2}{y}$, $\frac{\delta^2 u}{\delta x^2} = \frac{2}{y}\sec^2\frac{x^2}{y} + \frac{8x^2}{y^2}\sin\frac{x^2}{y}\sec^3\frac{x^2}{y}$, $\frac{\delta^2 u}{\delta x \delta y} = -\frac{2x}{y^2}\sec^2\frac{x^2}{y} - \frac{4x^3}{y^3}\sin\frac{x^2}{y}\sec^3\frac{x^2}{y}$, $\frac{\delta^2 u}{\delta y^2} = \frac{2x^2}{y^3}\sec^2\frac{x^2}{y} + \frac{2x^4}{y^4}\sin\frac{x^2}{y}\sec^3\frac{x^2}{y}$. **3220.** $\frac{\delta u}{\delta x} = yx^{y-1}$, $\frac{\delta u}{\delta y} = x^y\ln x$, $\frac{\delta^2 u}{\delta x^2} = y(y-1)x^{y-2}$, $\frac{\delta^2 u}{\delta x \delta y} = x^{y-1}(1+y\ln x)$, $\frac{\delta^2 u}{\delta y^2} = x^y\ln^2 x$ $(x>0)$. **3221.** $\frac{\delta u}{\delta x} = \frac{1}{x+y^2}$, $\frac{\delta u}{\delta y} = \frac{2y}{x+y^2}$, $\frac{\delta^2 u}{\delta x^2} = -\frac{1}{(x+y^2)^2}$, $\frac{\delta^2 u}{\delta x \delta y} = -\frac{2y}{(x+y^2)^2}$, $\frac{\delta^2 u}{\delta y^2} = \frac{2(x-y^2)}{(x+y^2)^2}$. **3222.** $\frac{\delta u}{\delta x} = -\frac{y}{x^2+y^2}$, $\frac{\delta u}{\delta y} = \frac{x}{x^2+y^2}$, $\frac{\delta^2 u}{\delta x^2} = \frac{2xy}{(x^2+y^2)^2}$, $\frac{\delta^2 u}{\delta x \delta y} = -\frac{x^2-y^2}{(x^2+y^2)^2}$, $\frac{\delta^2 u}{\delta y^2} = -\frac{2xy}{(x^2+y^2)^2}$. **3223.** $\frac{\delta u}{\delta x} = \frac{1}{1+x^2}$, $\frac{\delta u}{\delta y} = \frac{1}{1-y^2}$, $\frac{\delta^2 u}{\delta x^2} = -\frac{2x}{(1+x^2)^2}$, $\frac{\delta^2 u}{\delta x \delta y} = 0$, $\frac{\delta^2 u}{\delta y^2} = -\frac{2y}{(1+y^2)^2}$ $(xy \neq 1)$.

3224. $\frac{\delta u}{\delta x} = \frac{|y|}{x^2+y^2}$, $\frac{\delta u}{\delta y} = -\frac{x\operatorname{sgn} y}{x^2+y^2}$, $\frac{\delta^2 u}{\delta x^2} = -\frac{2x|y|}{(x^2+y^2)^2}$, $\frac{\delta^2 u}{\delta x \delta y} = \frac{(x^2-y^2)\operatorname{sgn} y}{(x^2+y^2)^2}$, $\frac{\delta^2 u}{\delta y^2} = \frac{2x|y|}{(x^2+y^2)^2}$ $(y \neq 0)$.

3225. $\frac{\delta u}{\delta x} = -\frac{x}{(x^2+y^2+z^2)^{3/2}}$, $\frac{\delta^2 u}{\delta x^2} = \frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{5/2}}$, $\frac{\delta^2 u}{\delta x \delta y} = \frac{3xy}{(x^2+y^2+z^2)^{5/2}}$. **3226.** $\frac{\delta u}{\delta x} = \frac{z}{x}\left(\frac{x}{y}\right)^z$, $\frac{\delta u}{\delta y} = -\frac{z}{y}\left(\frac{x}{y}\right)^z$, $\frac{\delta u}{\delta z} = \left(\frac{x}{y}\right)^z\ln\frac{x}{y}$, $\frac{\delta^2 u}{\delta x^2} = \frac{z(z-1)}{x^2}\left(\frac{x}{y}\right)^z$, $\frac{\delta^2 u}{\delta y^2} = \frac{z(z+1)}{y^2}\left(\frac{x}{y}\right)^z$, $\frac{\delta^2 u}{\delta z^2} = \left(\frac{x}{y}\right)^z\ln^2\frac{x}{y}$, $\frac{\delta^2 u}{\delta x \delta y} = -\frac{z^2}{xy}\left(\frac{x}{y}\right)^z$, $\frac{\delta^2 u}{\delta x \delta z} = \frac{1}{x}\left(\frac{x}{y}\right)^z\left(1+z\ln\frac{x}{y}\right)$, $\frac{\delta^2 u}{\delta y \delta z} = -\frac{1}{y}\left(\frac{x}{y}\right)^z\left(1+z\ln\frac{x}{y}\right)$ $\left(\frac{x}{y}>0\right)$. **3227.** $\frac{\delta u}{\delta x} = \frac{yu}{xz}$, $\frac{\delta u}{\delta y} = \frac{u\ln x}{z}$, $\frac{\delta u}{\delta z} = -\frac{yu}{z^2}\ln x$,