není diferencovatelná v bodě $(0,0)$. **3212.2** Funkce není diferencovatelná v bodě $(0,0)$. **3212.3** Funkce je diferencovatelná v bodě $(0,0)$. **3213.** $\frac{\partial u}{\partial x}=4x^3-8xy^2$, $\frac{\partial u}{\partial y}=4y^3-8x^2y$, $\frac{\partial^2 u}{\partial x^2}=12x^2-8y^2$, $\frac{\partial^2 u}{\partial x\partial y}=-16xy$, $\frac{\partial^2 u}{\partial y^2}=12y^2-8x^2$. **3214.** $\frac{\partial u}{\partial x}=y+\frac{1}{y}$, $\frac{\partial u}{\partial y}=x-\frac{x}{y^2}$, $\frac{\partial^2 u}{\partial x^2}=0$, $\frac{\partial^2 u}{\partial x\partial y}=1-\frac{1}{y^2}$, $\frac{\partial^2 u}{\partial y^2}=\frac{2x}{y^3}$. **3215.** $\frac{\partial u}{\partial x}=\frac{1}{y^2}$, $\frac{\partial u}{\partial y}=-\frac{2x}{y^3}$, $\frac{\partial^2 u}{\partial x^2}=0$, $\frac{\partial^2 u}{\partial x\partial y}=-\frac{2}{y^3}$, $\frac{\partial^2 u}{\partial y^2}=\frac{6x}{y^4}$. **3216.** $\frac{\partial u}{\partial x}=\frac{y^2}{(x^2+y^2)^{3/2}}$, $\frac{\partial u}{\partial y}=-\frac{xy}{(x^2+y^2)^{3/2}}$, $\frac{\partial^2 u}{\partial x^2}=-\frac{3xy^2}{(x^2+y^2)^{5/2}}$, $\frac{\partial^2 u}{\partial x\partial y}=\frac{y(2x^2-y^2)}{(x^2+y^2)^{5/2}}$, $\frac{\partial^2 u}{\partial y^2}=-\frac{x(x^2-2y^2)}{(x^2+y^2)^{5/2}}$. **3217.** $\frac{\partial u}{\partial x}=\sin(x+y)+x\cos(x+y)$, $\frac{\partial u}{\partial y}=x\cos(x+y)$, $\frac{\partial^2 u}{\partial x^2}=2\cos(x+y)-x\sin(x+y)$, $\frac{\partial^2 u}{\partial x\partial y}=\cos(x+y)-x\sin(x+y)$, $\frac{\partial^2 u}{\partial y^2}=-x\sin(x+y)$. **3218.** $\frac{\partial u}{\partial x}=-\frac{2x\sin x^2}{y}$, $\frac{\partial u}{\partial y}=-\frac{\cos x^2}{y^2}$, $\frac{\partial^2 u}{\partial x^2}=-\frac{2\sin x^2+4x^2\cos x^2}{y}$, $\frac{\partial^2 u}{\partial x\partial y}=\frac{2x\sin x^2}{y^2}$, $\frac{\partial^2 u}{\partial y^2}=\frac{2\cos x^2}{y^3}$. **3219.** $\frac{\partial u}{\partial x}=\frac{2x}{y}\sec^2\frac{x^2}{y}$, $\frac{\partial u}{\partial y}=-\frac{x^2}{y^2}\sec^2\frac{x^2}{y}$, $\frac{\partial^2 u}{\partial x^2}=\frac{2}{y}\sec^2\frac{x^2}{y}+\frac{8x^2}{y^2}\sin\frac{x^2}{y}\sec^3\frac{x^2}{y}$, $\frac{\partial^2 u}{\partial x\partial y}=-\frac{2x}{y^2}\sec^2\frac{x^2}{y}-\frac{4x^3}{y^3}\sin\frac{x^2}{y}\sec^3\frac{x^2}{y}$, $\frac{\partial^2 u}{\partial y^2}=\frac{2x^2}{y^3}\sec^2\frac{x^2}{y}+\frac{2x^4}{y^4}\sin\frac{x^2}{y}\sec^3\frac{x^2}{y}$. **3220.** $\frac{\partial u}{\partial x}=yx^{y-1}$, $\frac{\partial u}{\partial y}=x^y\ln x$, $\frac{\partial^2 u}{\partial x^2}=y(y-1)x^{y-2}$, $\frac{\partial^2 u}{\partial x\partial y}=x^{y-1}(1+y\ln x)$, $\frac{\partial^2 u}{\partial y^2}=x^y\ln^2 x$ $(x>0)$. **3221.** $\frac{\partial u}{\partial x}=\frac{1}{x+y^2}$, $\frac{\partial u}{\partial y}=\frac{2y}{x+y^2}$, $\frac{\partial^2 u}{\partial x^2}=-\frac{1}{(x+y^2)^2}$, $\frac{\partial^2 u}{\partial x\partial y}=-\frac{2y}{(x+y^2)^2}$, $\frac{\partial^2 u}{\partial y^2}=\frac{2(x-y^2)}{(x+y^2)^2}$. **3222.** $\frac{\partial u}{\partial x}=-\frac{y}{x^2+y^2}$, $\frac{\partial u}{\partial y}=\frac{x}{x^2+y^2}$, $\frac{\partial^2 u}{\partial x^2}=\frac{2xy}{(x^2+y^2)^2}$, $\frac{\partial^2 u}{\partial x\partial y}=-\frac{x^2-y^2}{(x^2+y^2)^2}$, $\frac{\partial^2 u}{\partial y^2}=-\frac{2xy}{(x^2+y^2)^2}$. **3223.** $\frac{\partial u}{\partial x}=\frac{1}{1+x^2}$, $\frac{\partial u}{\partial y}=\frac{1}{1+y^2}$, $\frac{\partial^2 u}{\partial x^2}=-\frac{2x}{(1+x^2)^2}$, $\frac{\partial^2 u}{\partial x\partial y}=0$, $\frac{\partial^2 u}{\partial y^2}=-\frac{2y}{(1+y^2)^2}$ $(xy\neq 1)$.

3224. $\frac{\partial u}{\partial x}=\frac{|y|}{x^2+y^2}$, $\frac{\partial u}{\partial y}=-\frac{x\,\mathrm{sgn}\,y}{x^2+y^2}$, $\frac{\partial^2 u}{\partial x^2}=-\frac{2x|y|}{(x^2+y^2)^2}$, $\frac{\partial^2 u}{\partial x\partial y}=\frac{(x^2-y^2)\,\mathrm{sgn}\,y}{(x^2+y^2)^2}$, $\frac{\partial^2 u}{\partial y^2}=\frac{2x|y|}{(x^2+y^2)^2}$ $(y\neq 0)$.

3225. $\frac{\partial u}{\partial x}=-\frac{x}{(x^2+y^2+z^2)^{3/2}}$, $\frac{\partial^2 u}{\partial x^2}=\frac{2x^2-y^2-z^2}{(x^2+y^2+z^2)^{5/2}}$, $\frac{\partial^2 u}{\partial x\partial y}=\frac{3xy}{(x^2+y^2+z^2)^{5/2}}$. **3226.** $\frac{\partial u}{\partial x}=\frac{z}{x}\left(\frac{x}{y}\right)^z$, $\frac{\partial u}{\partial y}=-\frac{z}{y}\left(\frac{x}{y}\right)^z$, $\frac{\partial u}{\partial z}=\left(\frac{x}{y}\right)^z\ln\frac{x}{y}$, $\frac{\partial^2 u}{\partial x^2}=\frac{z(z-1)}{x^2}\left(\frac{x}{y}\right)^z$, $\frac{\partial^2 u}{\partial y^2}=\frac{z(z+1)}{y^2}\left(\frac{x}{y}\right)^z$, $\frac{\partial^2 u}{\partial z^2}=\left(\frac{x}{y}\right)^z\ln^2\frac{x}{y}$, $\frac{\partial^2 u}{\partial x\partial y}=-\frac{z^2}{xy}\left(\frac{x}{y}\right)^z$, $\frac{\partial^2 u}{\partial x\partial z}=\frac{1}{x}\left(\frac{x}{y}\right)^z\left(1+z\ln\frac{x}{y}\right)$, $\frac{\partial^2 u}{\partial y\partial z}=-\frac{1}{y}\left(\frac{x}{y}\right)^z\left(1+z\ln\frac{x}{y}\right)$ $\left(\frac{x}{y}>0\right)$. **3227.** $\frac{\partial u}{\partial x}=\frac{yu}{xz}$, $\frac{\partial u}{\partial y}=\frac{u\ln x}{z}$, $\frac{\partial u}{\partial z}=-\frac{yu}{z^2}\ln x$, $\frac{\partial^2 u}{\partial x^2}=\frac{y(y-z)u}{x^2z^2}$, $\frac{\partial^2 u}{\partial y^2}=\frac{u\ln^2 x}{z^2}$, $\frac{\partial^2 u}{\partial z^2}=\frac{yu\ln x}{z^4}(2z+y\ln x)$, $\frac{\partial^2 u}{\partial x\partial y}=\frac{(z+y\ln x)u}{xz^2}$, $\frac{\partial^2 u}{\partial x\partial z}=-\frac{yu(z+y\ln x)}{xz^3}$, $\frac{\partial^2 u}{\partial y\partial z}=-\frac{u\ln x(z+y\ln x)}{z^3}$ $(xz\neq 0)$. **3228.** $\frac{\partial u}{\partial x}=\frac{y^z}{x}u$, $\frac{\partial u}{\partial y}=zy^{z-1}u\ln x$, $\frac{\partial u}{\partial z}=y^z u\ln x\ln y$, $\frac{\partial^2 u}{\partial x^2}=\frac{y^z(y^z-1)}{x^2}u$, $\frac{\partial^2 u}{\partial y^2}=zy^{z-2}u(z-1+zy^z\ln x)\ln x$, $\frac{\partial^2 u}{\partial z^2}=y^zu(1+y^z\ln x)\ln x\ln^2 y$, $\frac{\partial^2 u}{\partial x\partial y}=\frac{zy^{z-1}u}{x}(1+y^z\ln x)$, $\frac{\partial^2 u}{\partial x\partial z}=\frac{y^zu\ln y}{x}(1+y^z\ln x)$, $\frac{\partial^2 u}{\partial y\partial z}=y^{z-1}u\ln x\left[1+z\ln y(1+y^z\ln x)\right]$ $(x>0,\ y>0)$. **3230.1** $f''_{xy}(0,0)$ neexistuje.

3235. $du=x^{m-1}y^{n-1}(my\,dx+nx\,dy)$, $d^2u=x^{m-2}y^{n-2}\left[m(m-1)y^2dx^2+2mn\,xy\,dx\,dy+n(n-1)x^2dy^2\right]$.

3236. $du=\frac{y\,dx-x\,dy}{y^2}$, $d^2u=-\frac{2}{y^3}dy(y\,dx-x\,dy)$. **3237.** $du=\frac{x\,dx+y\,dy}{\sqrt{x^2+y^2}}$, $d^2u=\frac{(y\,dx-x\,dy)^2}{(x^2+y^2)^{3/2}}$.

3238. $du=\frac{x\,dx+y\,dy}{x^2+y^2}$, $d^2u=\frac{(y^2-x^2)(dx^2-dy^2)-4xy\,dx\,dy}{(x^2+y^2)^2}$. **3239.** $du=e^{xy}(y\,dx+x\,dy)$;

$d^2u=e^{xy}\left[y^2dx^2+2(1+xy)\,dxdy+x^2dy^2\right]$. **3240.** $du=(y+z)dx+(z+x)dy+(x+y)dz$; $d^2u=2(dxdy+dydz+dzdx)$.
3241. $du=\dfrac{(x^2+y^2)dz-2z(xdx+ydy)}{(x^2+y^2)^2}$, $d^2u=\dfrac{2z\left[(3x^2-y^2)dx^2+8xydxdy+(3y^2-x^2)dy^2\right]-4(x^2+y^2)(xdx+ydy)dz}{(x^2+y^2)^3}$.
3242. $dx-dy$, $-2(dx-dy)(dy+dz)$. **3244.** a) $1+mx+ny$; b) xy; c) $x+y$. **3245.** a) 108,972; b) 1,055; c) 2,95; d) 0,502; e) 0,97. **3246.** Úhlopříčka se zmenší přibližně o 3 mm; plocha se zmenší přibližně o 140 cm^2. **3247.** O 1,7 mm. **3249.** $\Delta\approx 10{,}2\ m^3$; $\delta\approx 13\ \%$. **3250.** $\Delta\approx 7{,}6$ m. **3251.** $f'_x(x,y)$ a $f'_y(x,y)$ jsou neomezené v okolí bodu $(0,0)$. **3256.** $\dfrac{\partial^4u}{\partial x^4}=24$, $\dfrac{\partial^4u}{\partial x^3\partial y}=0$, $\dfrac{\partial^4u}{\partial x^2\partial y^2}=-16$. **3257.** $\dfrac{\partial^3u}{\partial x^2\partial y}=0$.

3258. $\dfrac{\partial^6u}{\partial x^3\partial y^3}=-6(\cos x+\cos y)$. **3259.** $\dfrac{\partial^3u}{\partial x\partial y\partial z}=0$. **3260.** $\dfrac{\partial^3u}{\partial x\partial y\partial z}=e^{xyz}\left(1+3xyz+x^2y^2z^2\right)$.

3261. $\dfrac{\partial^4u}{\partial x\partial y\partial\xi\partial\eta}=-\dfrac{6}{r^4}+\dfrac{48(x-\xi)^2(y-\eta)^2}{r^8}$, kde $r=\sqrt{(x-\xi)^2+(y-\eta)^2}$. **3262.** $\dfrac{\partial^{p+q}u}{\partial x^p\partial x^q}=p!\,q!$.

3263. $\dfrac{2(-1)^m(m+n-1)!\,(nx+my)}{(x+y)^{m+n+1}}$. **3264.** $e^{x+y}\left[x^2+y^2+2(mx+ny)+m(m-1)+n(n-1)\right]$.

3265. $(x+p)(y+q)(z+r)e^{x+y+z}$. **3266.** $\sin\dfrac{n\pi}{2}$. **3267.** $F(t)=f'(t)+3tf''(t)+t^2f'''(t)$.

3268. $d^4u=24\left(dx^4-2dx^3dy-2dxdy^3+dy^4\right)$; $\dfrac{\partial^4u}{\partial x^4}=24$, $\dfrac{\partial^4u}{\partial x^3\partial y}=-12$, $\dfrac{\partial^4u}{\partial x^2\partial y^2}=0$, $\dfrac{\partial^4u}{\partial x\partial y^3}=-12$, $\dfrac{\partial^4u}{\partial y^4}=24$.

3269. $d^3u=6\left(dx^3-3dx^2dy+3dxdy^2+dy^3\right)$.

3270. $d^3u=-8(xdx+ydy)^3\cos(x^2+y^2)-12(xdx+ydy)(dx^2+dy^2)\sin(x^2+y^2)$. **3271.** $d^{10}u=-\dfrac{9!\,(dx+dy)^{10}}{(x+y)^{10}}$.

3272. $d^6u=-(dx^6-15dx^4dy^2+15dx^2dy^4-dy^6)\cos x\cosh y-2dxdy(3dx^4-10dx^2dy^2+3dy^4)\sin x\sinh y$.

3273. $d^3u=6dxdydz$. **3274.** $d^4u=2\left(\dfrac{dx^4}{x^3}+\dfrac{dy^4}{y^3}+\dfrac{dz^4}{z^3}\right)$. **3275.** $d^nu=e^{ax+by}(adx+bdy)^n$.

3276. $d^nu=\displaystyle\sum_{k=0}^{n}\binom{n}{k}X^{(n-k)}(x)Y^{(k)}(y)dx^{n-k}dy^k$. **3277.** $d^nu=f^{(n)}(x+y+z)\,(dx+dy+dz)^n$.

3278. $d^nu=e^{ax+by+cz}(adx+bdy+cdz)^n$. **3280.** a) $Au=-u$, $A^2u=u$; b) $Au=1$, $A^2u=0$. **3281.** a) $\Delta u=0$; b) $\Delta u=0$. **3282.** a) $\Delta_1u=9\left[(x^2-yz)^2+(y^2-xz)^2+(z^2-xy)^2\right]$, $\Delta_2u=6(x+y+z)$; b) $\Delta_1u=\dfrac{1}{r^4}$, kde $r=\sqrt{x^2+y^2+z^2}$, $\Delta_2u=0$. **3283.** $\dfrac{\partial u}{\partial x}=2xf'(x^2+y^2+z^2)$; $\dfrac{\partial^2u}{\partial x^2}=2f'(x^2+y^2+z^2)+4x^2f''(x^2+y^2+z^2)$; $\dfrac{\partial^2u}{\partial x\partial y}=4xyf''(x^2+y^2+z^2)$.

3284. $\dfrac{\partial u}{\partial x}=f_1'\left(x,\dfrac{x}{y}\right)+\dfrac{1}{y}f_2'\left(x,\dfrac{x}{y}\right)$; $\dfrac{\partial u}{\partial y}=-\dfrac{x}{y^2}f_2'\left(x,\dfrac{x}{y}\right)$; $\dfrac{\partial^2u}{\partial x^2}=f_{11}''\left(x,\dfrac{x}{y}\right)+\dfrac{2}{y}f_{12}''\left(x,\dfrac{x}{y}\right)+\dfrac{1}{y^2}f_{22}''\left(x,\dfrac{x}{y}\right)$;
$\dfrac{\partial^2u}{\partial x\partial y}=-\dfrac{x}{y^2}f_{12}''\left(x,\dfrac{x}{y}\right)-\dfrac{x}{y^3}f_{22}''\left(x,\dfrac{x}{y}\right)-\dfrac{1}{y^2}f_2'\left(x,\dfrac{x}{y}\right)$; $\dfrac{\partial^2u}{\partial y^2}=\dfrac{x^2}{y^4}f_{22}''\left(x,\dfrac{x}{y}\right)+\dfrac{2x}{y^3}f_2'\left(x,\dfrac{x}{y}\right)$.

3285. $\dfrac{\partial u}{\partial x}=f_1'+yf_2'+yzf_3'$; $\dfrac{\partial u}{\partial y}=xf_2'+xzf_3'$; $\dfrac{\partial u}{\partial z}=xyf_3'$; $\dfrac{\partial^2u}{\partial x^2}=f_{11}''+y^2f_{22}''+y^2z^2f_{33}''+2yf_{12}''+2yzf_{13}''+2y^2zf_{23}''$;
$\dfrac{\partial^2u}{\partial y^2}=x^2f_{22}''+2x^2zf_{23}''+x^2z^2f_{33}''$; $\dfrac{\partial^2u}{\partial z^2}=x^2y^2f_{33}''$; $\dfrac{\partial^2u}{\partial x\partial y}=xyf_{22}''+xyz^2f_{33}''+xf_{12}''+xzf_{13}''+2xyzf_{23}''+f_2'+zf_3'$;
$\dfrac{\partial^2u}{\partial x\partial z}=xyf_{13}''+xy^2f_{23}''+xy^2zf_{33}''+yf_3'$; $\dfrac{\partial^2u}{\partial y\partial z}=x^2yf_{23}''+x^2yzf_{33}''+xf_3'$.

3286. $\dfrac{\partial^2u}{\partial x\partial y}=f_{11}''+(x+y)f_{12}''+xyf_{22}''+f_2'$. **3287.** $\Delta u=3f_{11}''+4(x+y+z)f_{12}''+4(x^2+y^2+z^2)f_{22}''+6f_2$.

3288. $du=f'(t)(dx+dy)$; $d^2u=f''(t)(dx+dy)^2$. **3289.** $du=f'(t)\dfrac{xdy-ydx}{x^2}$;