CVIČENÍ Z MATEMATICKÉ ANALÝZY 1

Rekurentně zadané posloupnosti a Hromadné hodnoty

1. Spočtěte limity posloupnosti $\{a_n\}$ jestliže

1) $a_1 = \sqrt{2}, a_{n+1} = \sqrt{a_n + 2}$,

2) $a_1, a_2 \in \mathbb{R}, a_1 < a_2, a_{n+1} = \dfrac{a_n + a_{n-1}}{2}$,

3) $a_1 = 10, a_{n+1} = 6 - \dfrac{5}{a_n}$,

4) $a_1 = 0, a_{n+1} = \dfrac{a_n + 3}{4}$,

5) $a_1 \in (0, \infty), a_{n+1} = \dfrac{1}{2}\left(a_n + \dfrac{1}{a_n}\right)$,

6) $a \in [0, 1], a_1 = 0, a_{n+1} = a_n + \dfrac{1}{2}(a - a_n^2)$.

2. Je dána posloupnost $\{a_n\}$. Nalezněte $\limsup a_n$, $\liminf a_n$ a $H(a_n)$, jestliže

1) $a_n = (-1)^n$,

2) $a_n = \sqrt[n]{4^{(-1)^n} + 2}$,

3) $a_n = \dfrac{(-1)^n}{n} + \dfrac{1 - (-1)^n}{2}$,

4) $a_n = (-1)^n\left(2 + \dfrac{3}{n}\right)$,

5) $a_n = 1 + \dfrac{n}{n + 1}\cos\left(\dfrac{n\pi}{2}\right)$,

6) $a_n = \dfrac{1}{2}\left(n - 2 - 3\left\lfloor\dfrac{n-1}{3}\right\rfloor\right)\left(n - 3 - 3\left\lfloor\dfrac{n-1}{3}\right\rfloor\right)$,

7) $a_n = \dfrac{(1 + \cos(n\pi))\log 3n + \log n}{\log 2n}$,

8) $a_n = \left(1 + \dfrac{1}{n}\right)^n (-1)^n + \sin\left(\dfrac{n\pi}{4}\right)$,

9) $a_n = \dfrac{2n^2 + 2n + n\sin 2n}{n\cos 3n + (2n + \sin 4n)^2}$,

10) $a_n = 1 + 2(-1)^{n+1} + 3(-1)^{\frac{n(n-1)}{2}}$,

11) $a_n = (\cos \pi n)n$,

12) $a_n = -n(2 + (-1)^n)$,

13) $a_n = \left\{\dfrac{1}{2}, \dfrac{1}{3}, \dfrac{2}{3}, \dfrac{1}{4}, \dfrac{2}{4}, \dfrac{3}{4}, \dfrac{1}{5}, \ldots, \dfrac{1}{n}, \dfrac{2}{n}, \ldots, \dfrac{n-1}{n}, \dfrac{1}{n+1}, \ldots\right\}$.