An introduction to conic optimization

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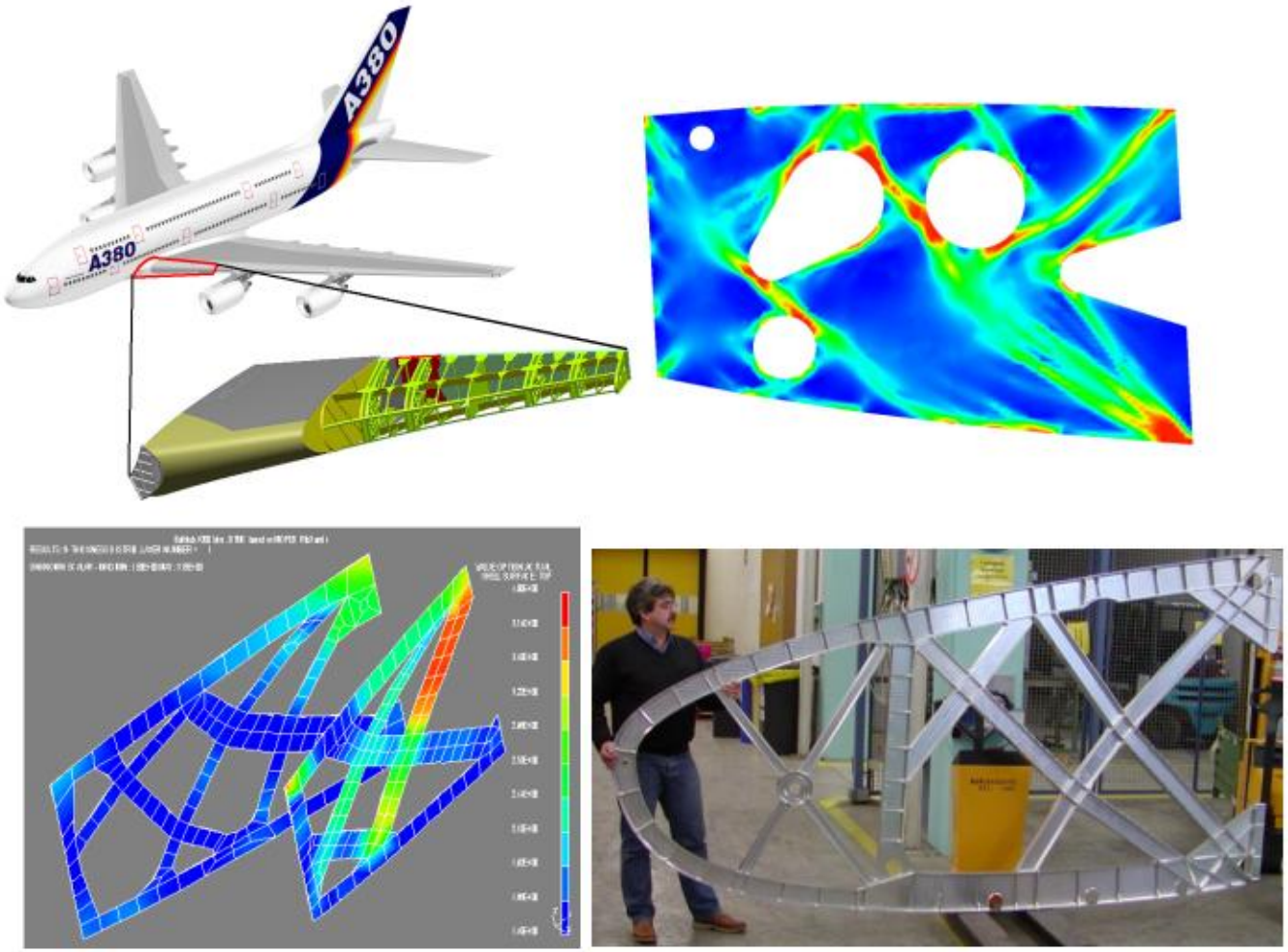
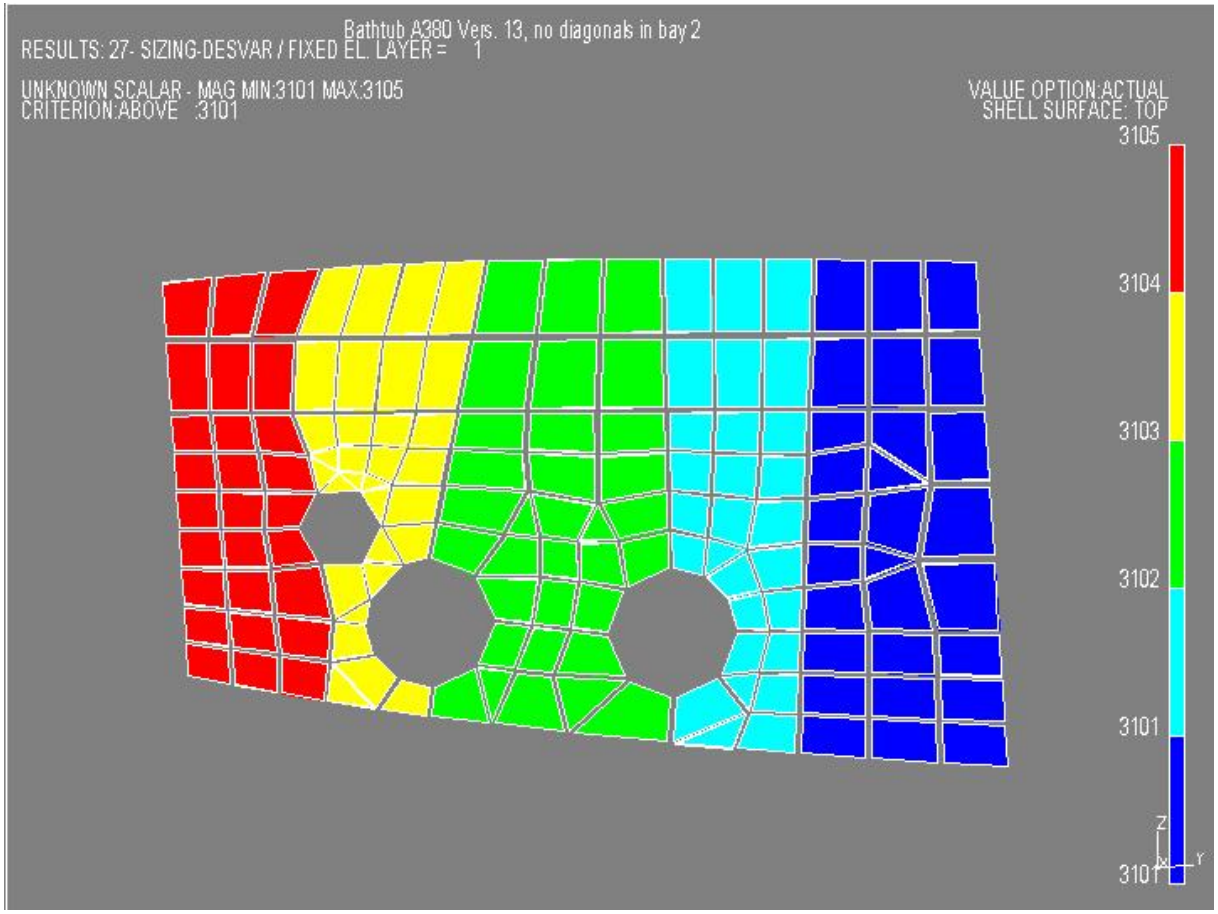


Design of ribs in the leading edge of Airbus A380

About 15-20% weight savings compared to traditional design



Michal Kočvara, Dept Math, Univ Birmingham

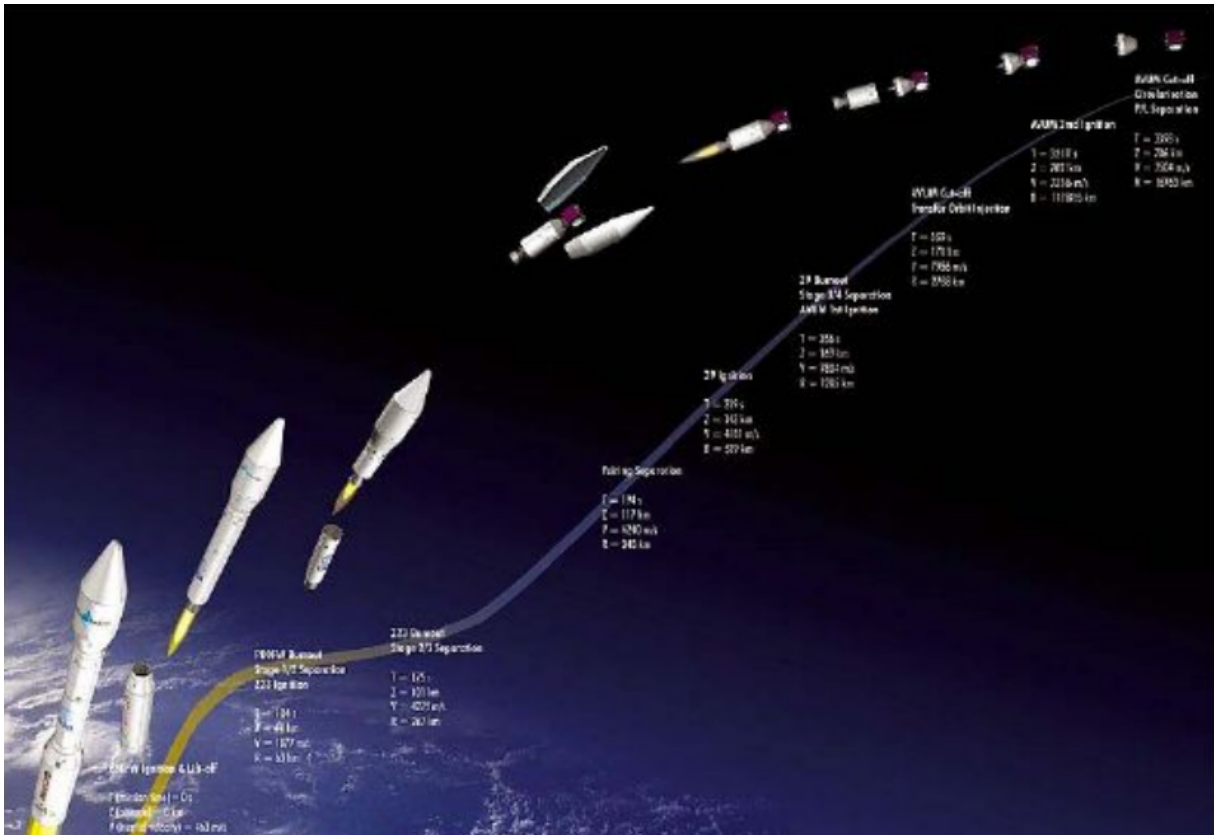
Design of control laws for Ariane 5

Wind disturbance rejection

Heavy load flexible structure

H_∞ robust control and optimization

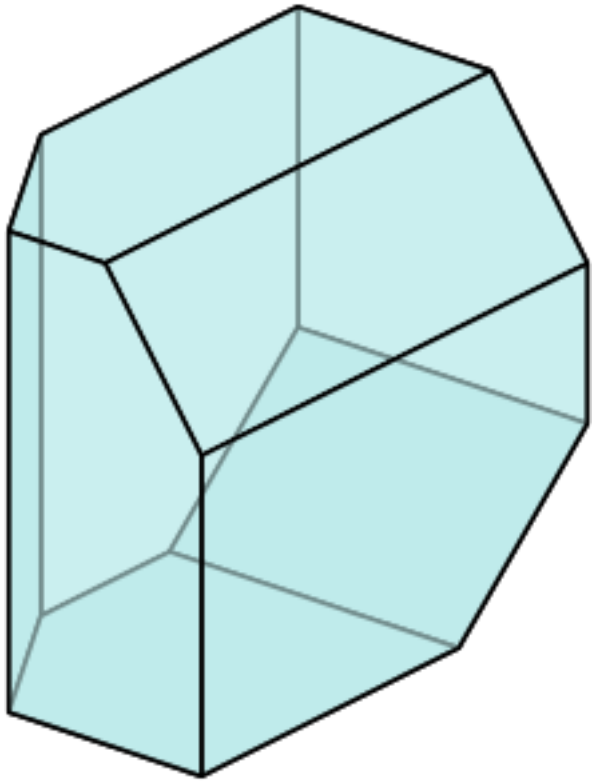
Validation and verification for Vega launcher



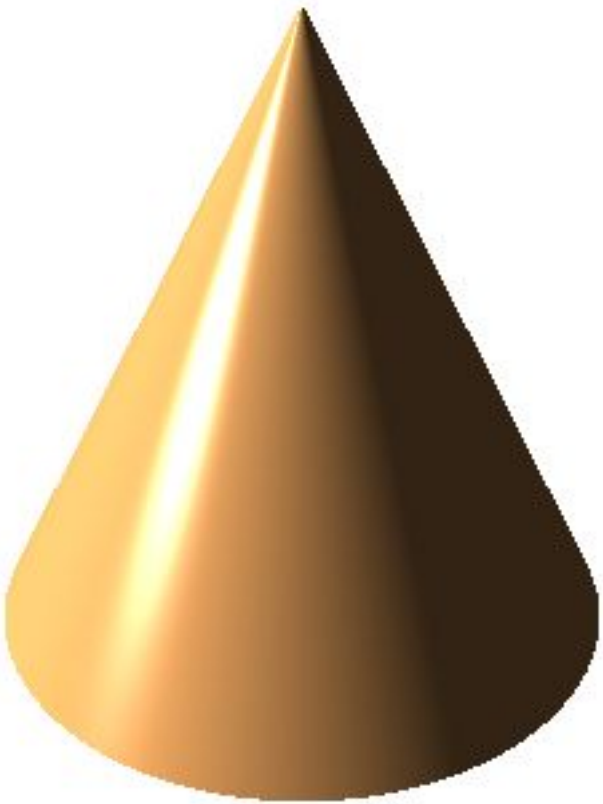
Conic optimization for assessing stability and performance

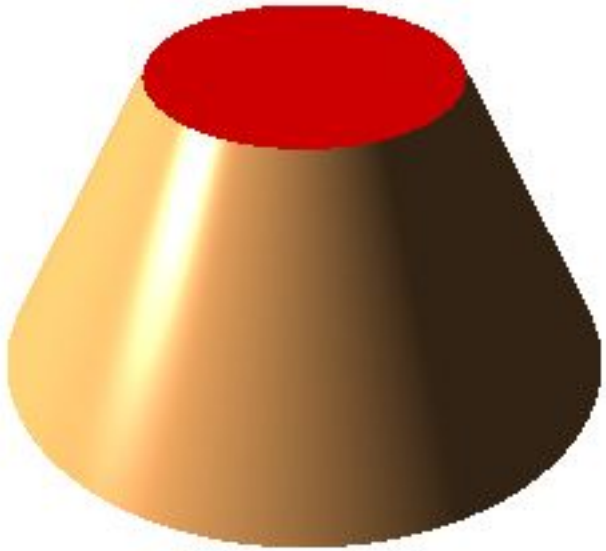
Convex conic geometry

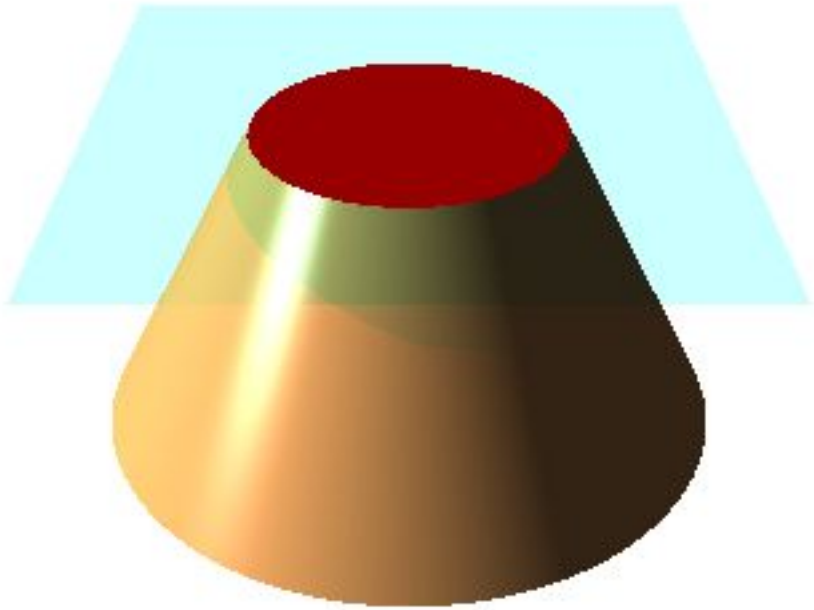
Polytopes

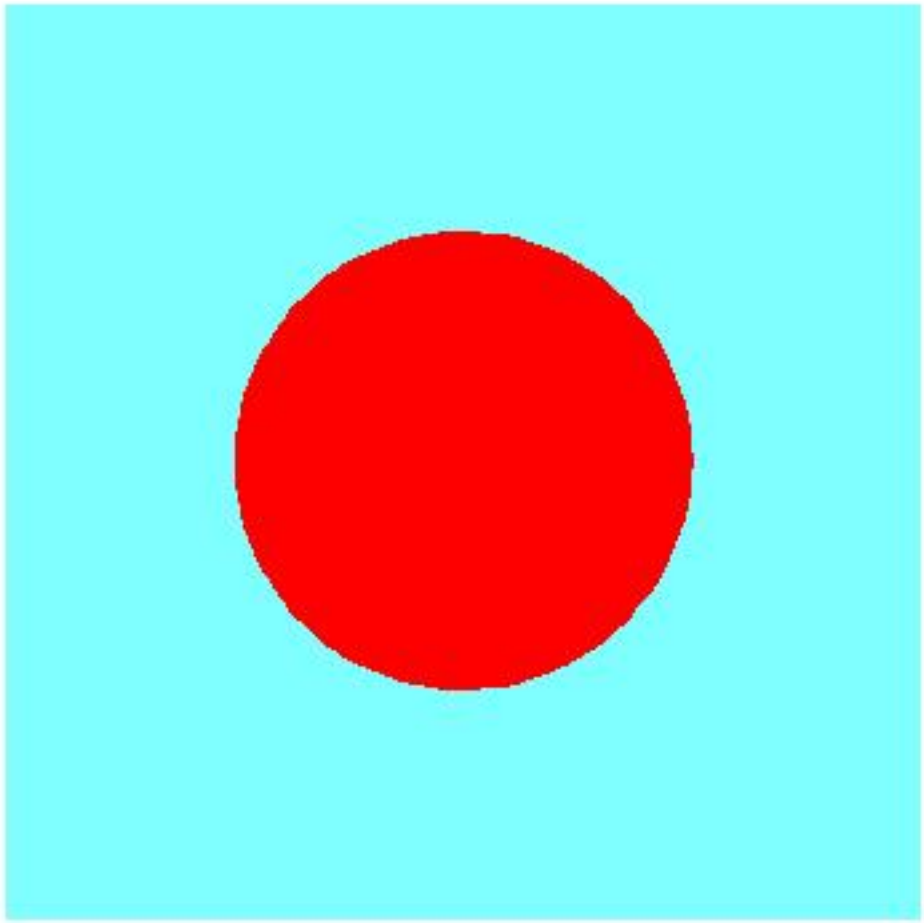


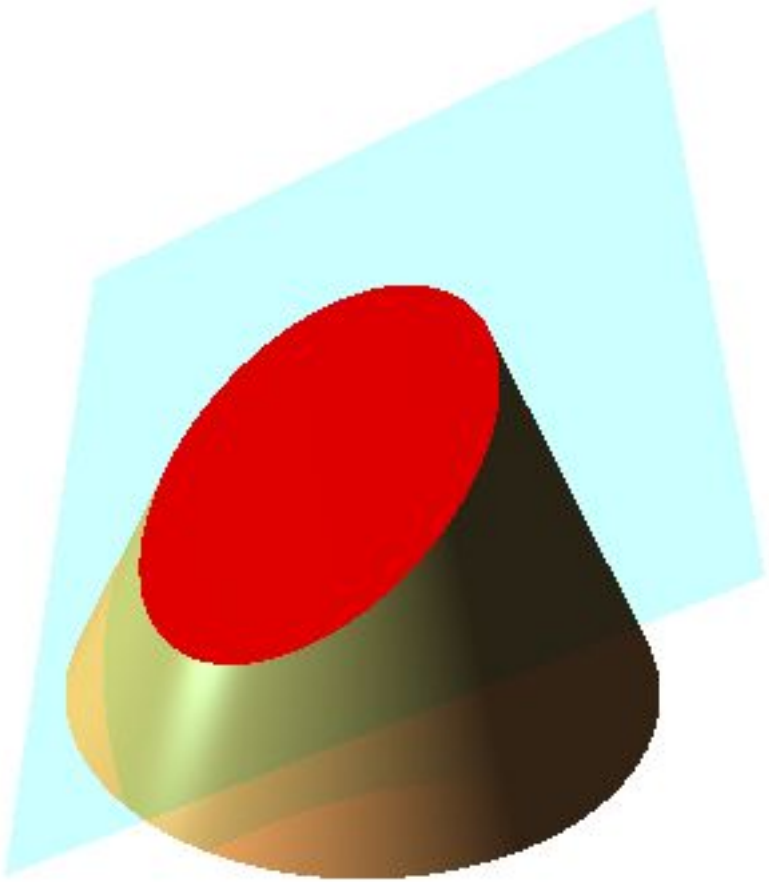
Conics

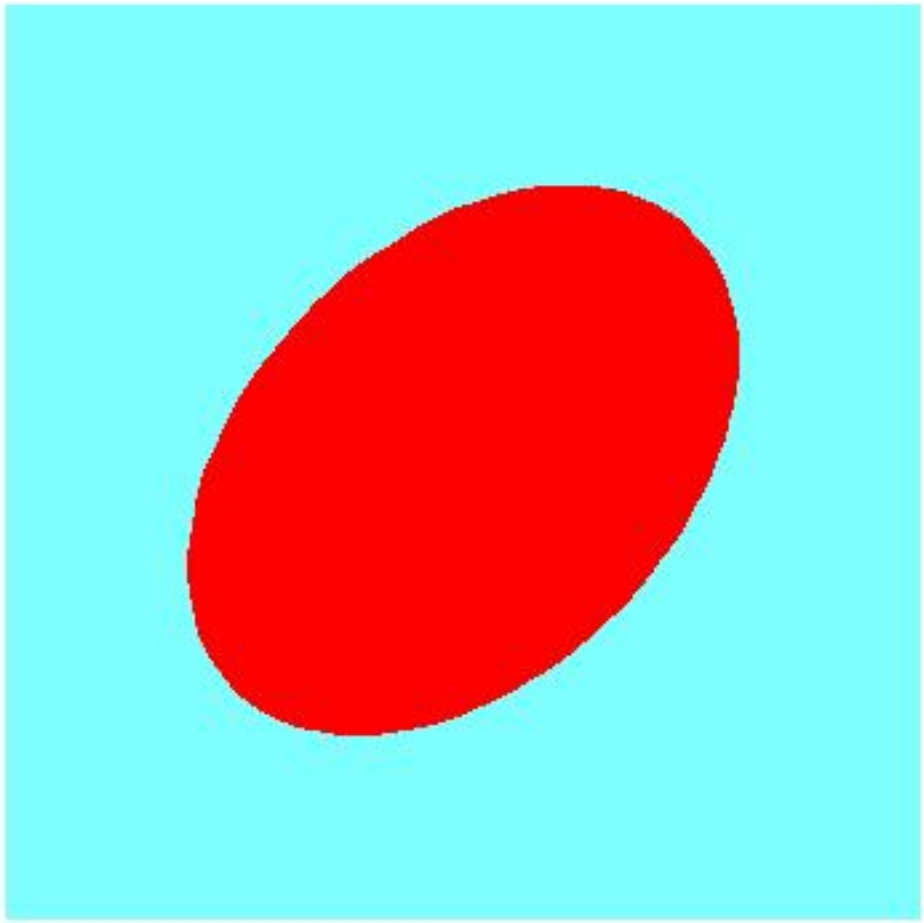


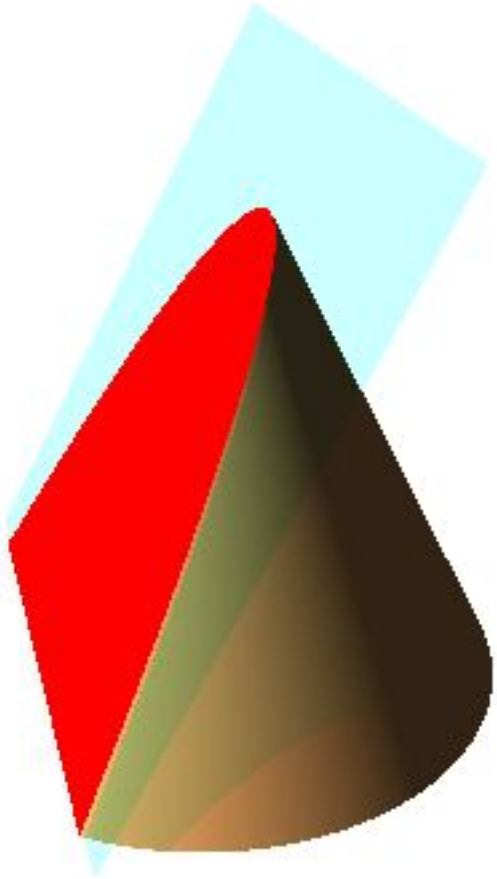


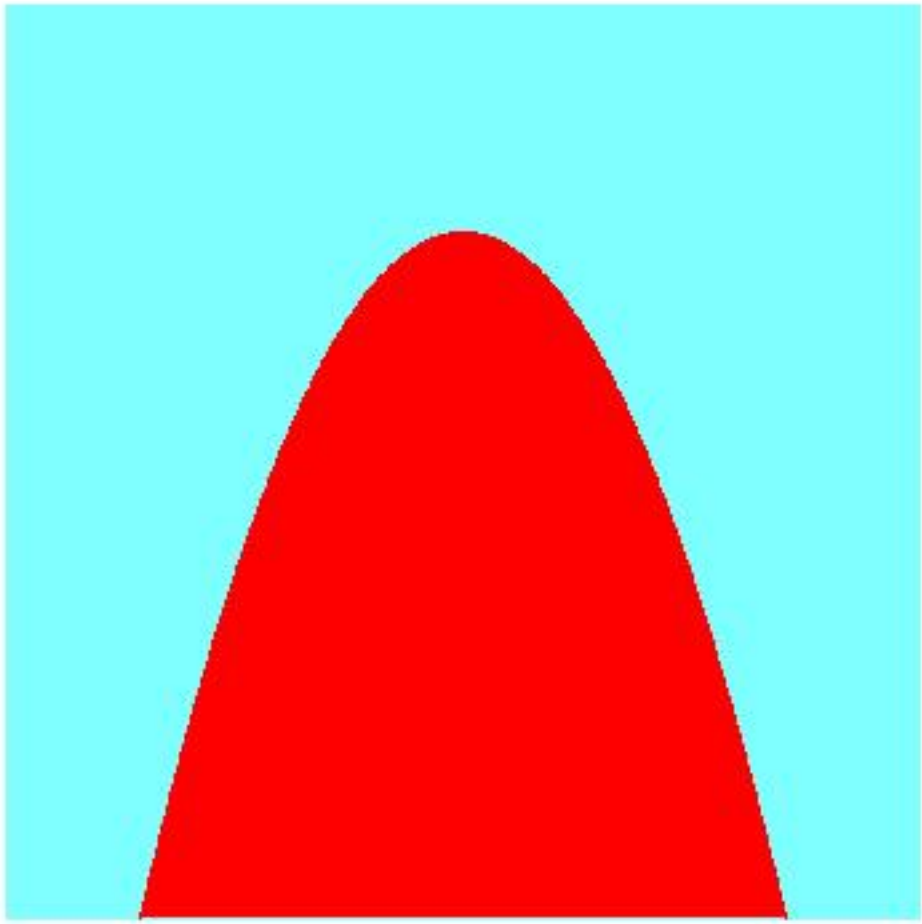








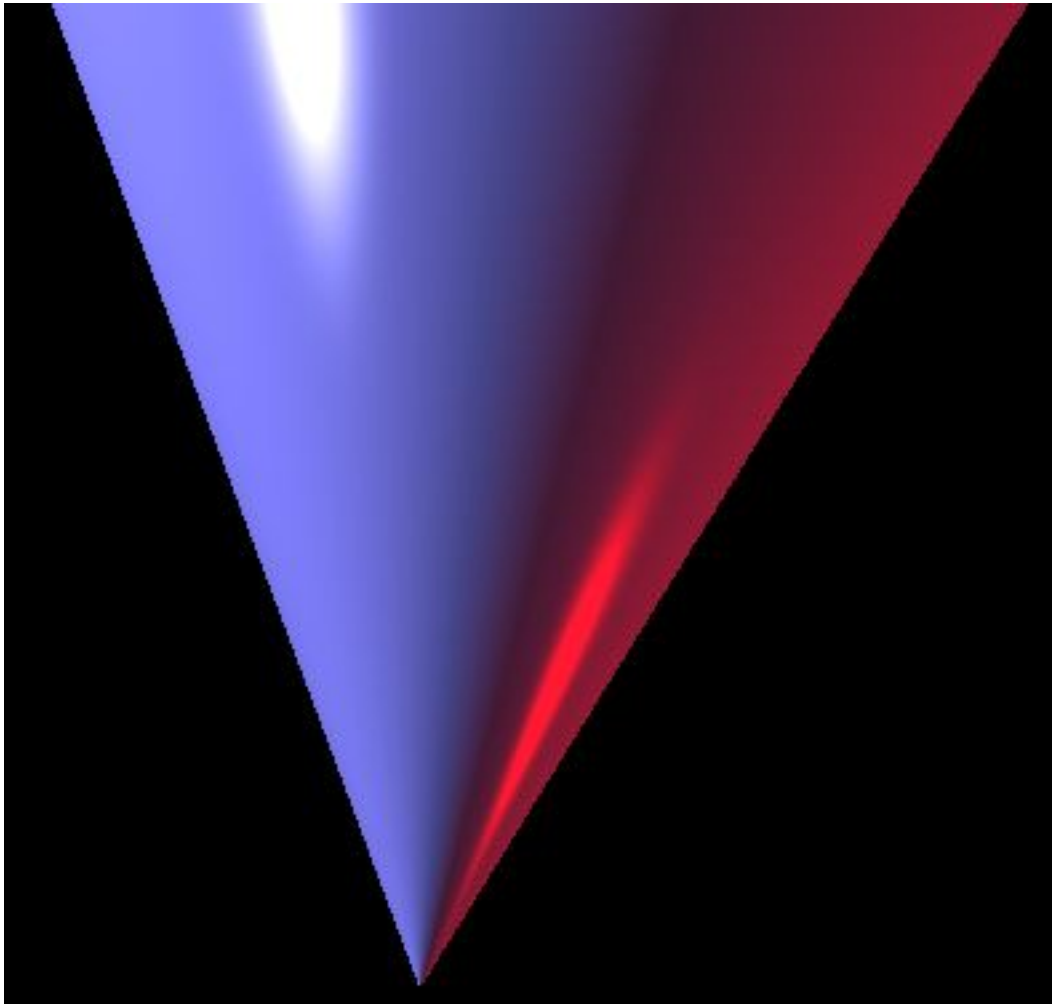


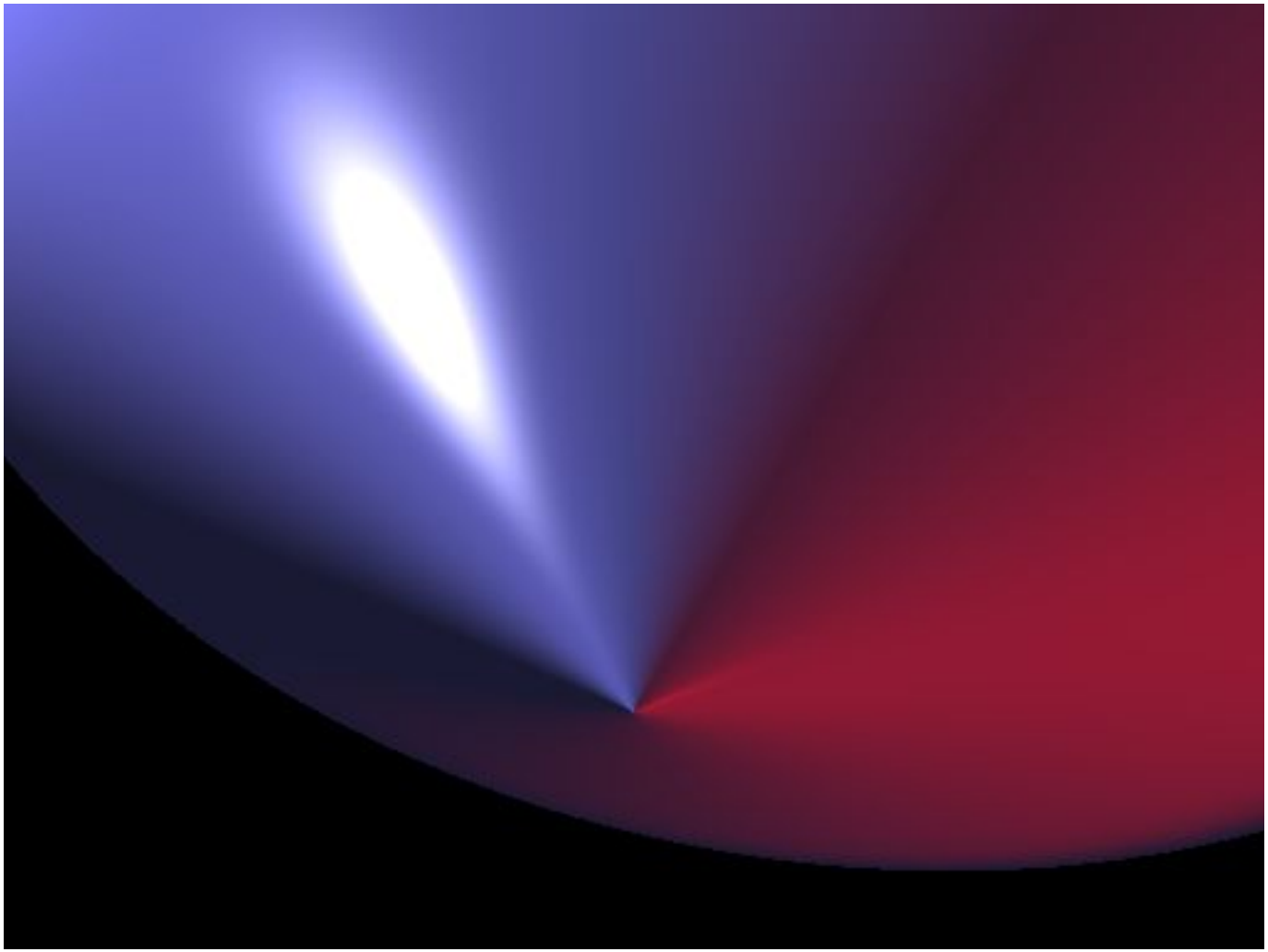


$$x_1 y^2 + x_2 y + x_3 \geq 0 \quad \forall y \in \mathbb{R}$$

$$\Longleftrightarrow$$

$$\begin{aligned} x_1 &\geq 0 \\ 4x_1 x_3 &\geq x_2^2 \end{aligned}$$



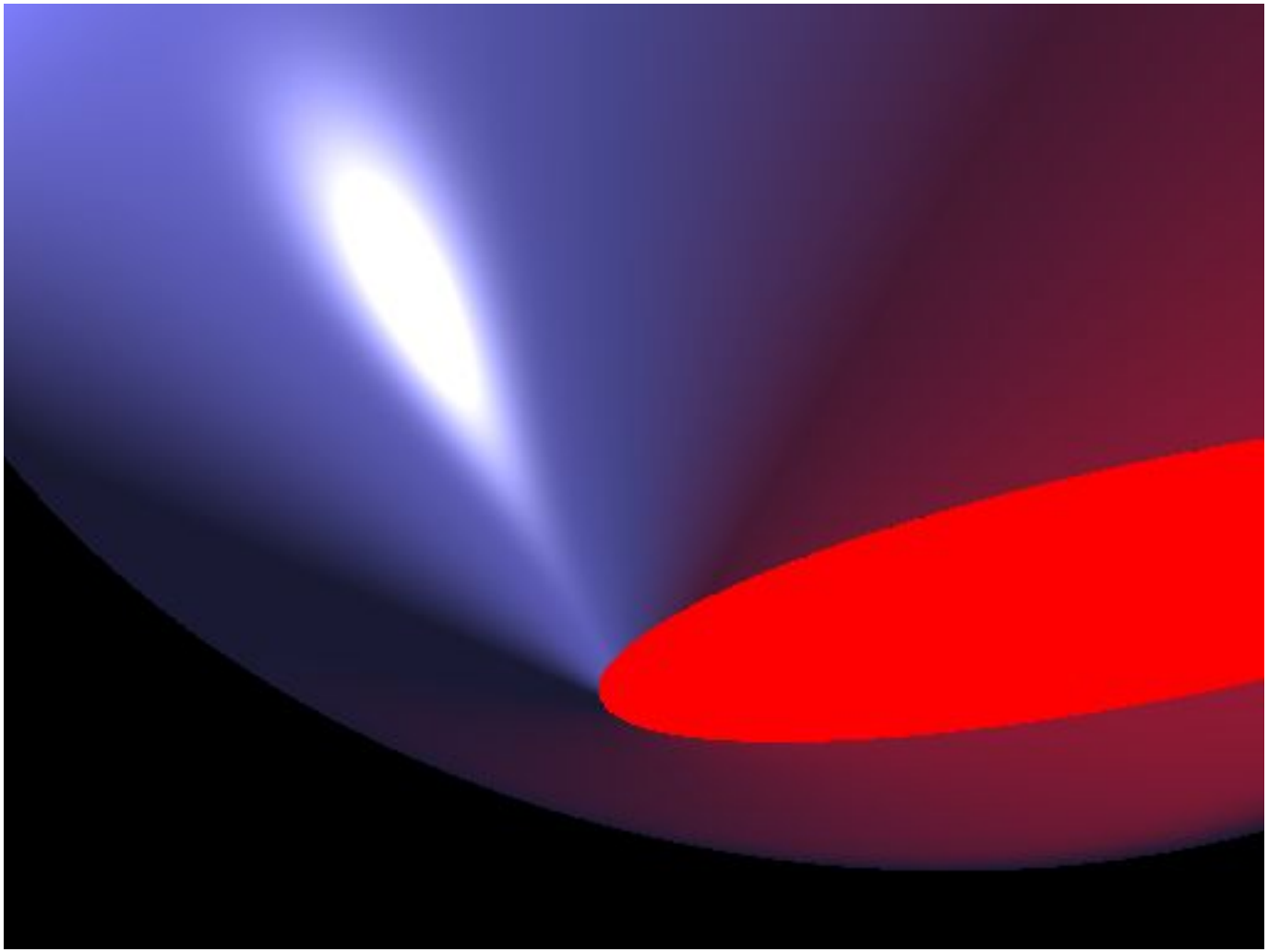


Intersection of convex cone

$$x_1 \geq 0$$
$$4x_1x_3 \geq x_2^2$$

with affine subspace

$$x_1 = 1$$



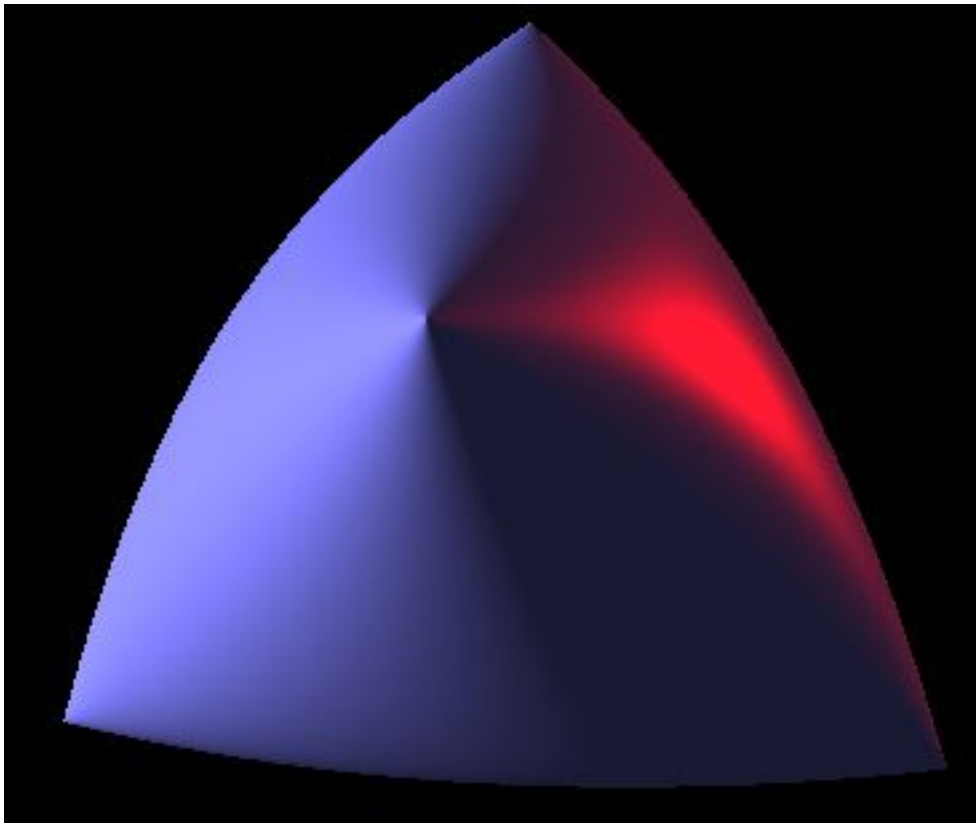
Spectrahedra

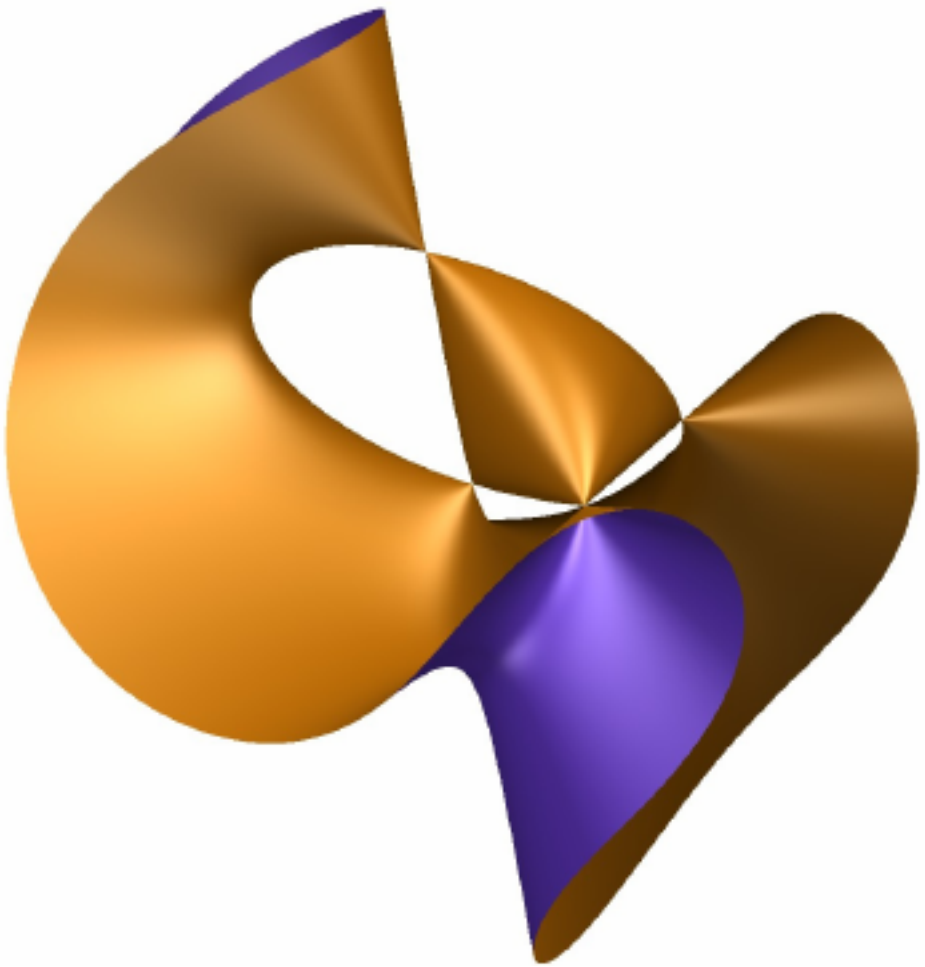
$$x_1 y_1^2 + x_2 y_1 + x_3 + x_4 y_2 + x_5 y_1 y_2 + x_6 y_2^2 \geq 0 \quad \forall y \in \mathbb{R}^2$$

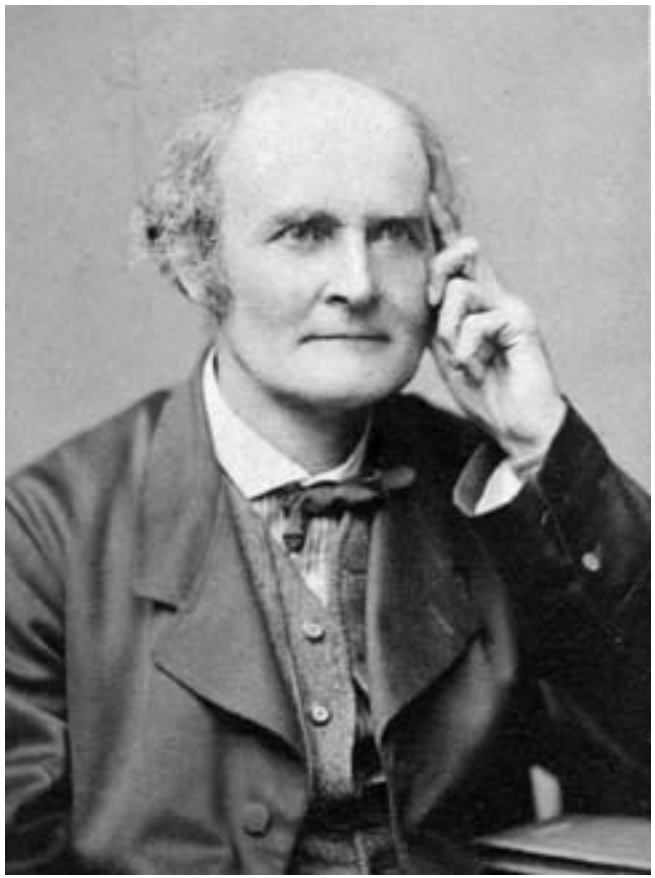
$$y_1^2 + x_2 y_1 + 1 + x_4 y_2 + x_5 y_1 y_2 + y_2^2 \geq 0 \quad \forall y \in \mathbb{R}^2$$

$$\Longleftrightarrow$$

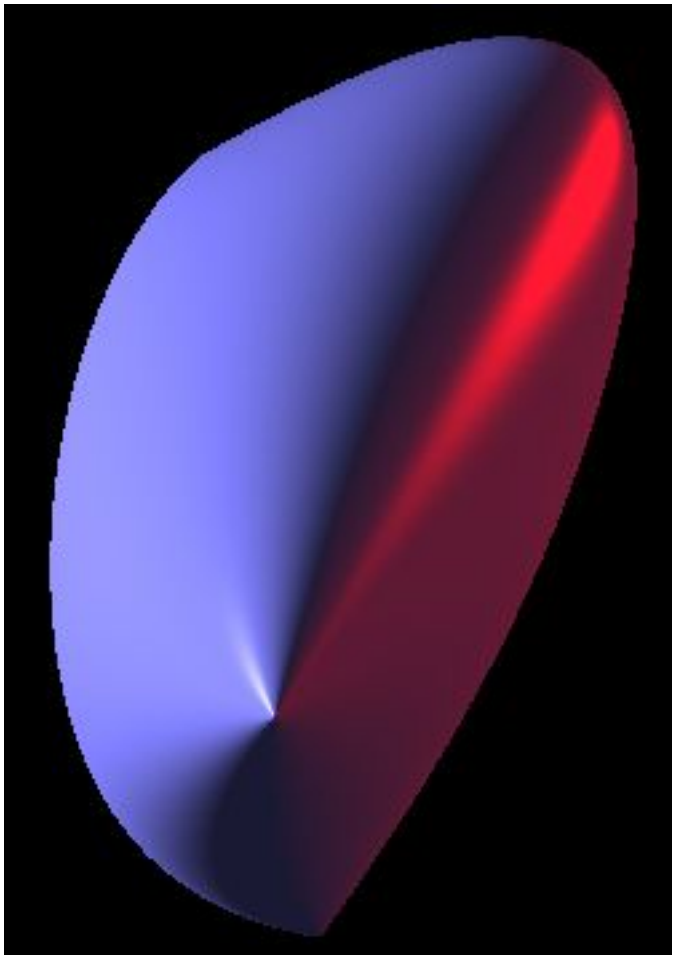
$$4 + x_2 x_4 x_5 \geq x_2^2 + x_4^2 + x_5^2$$
$$12 \geq x_2^2 + x_4^2 + x_5^2$$

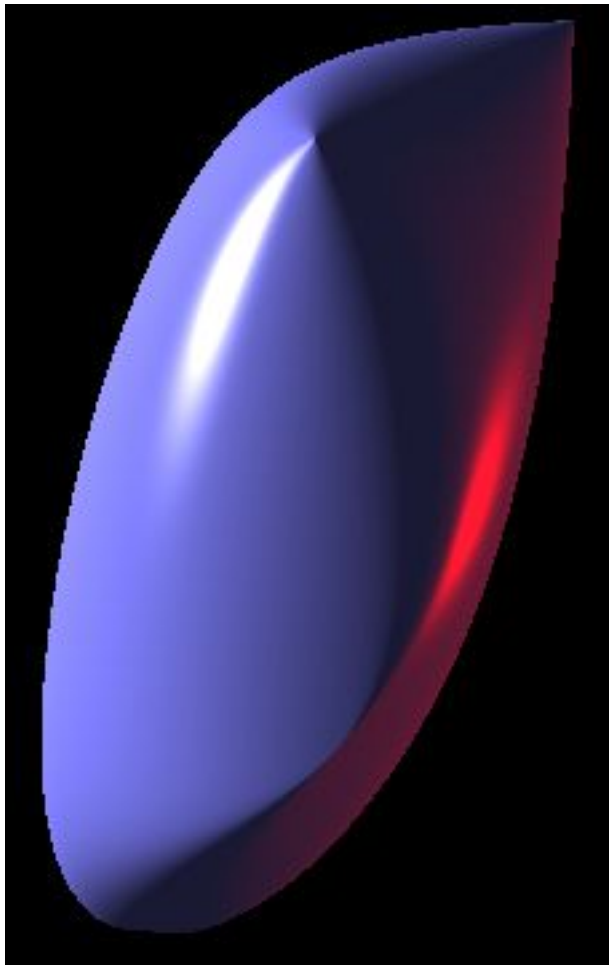


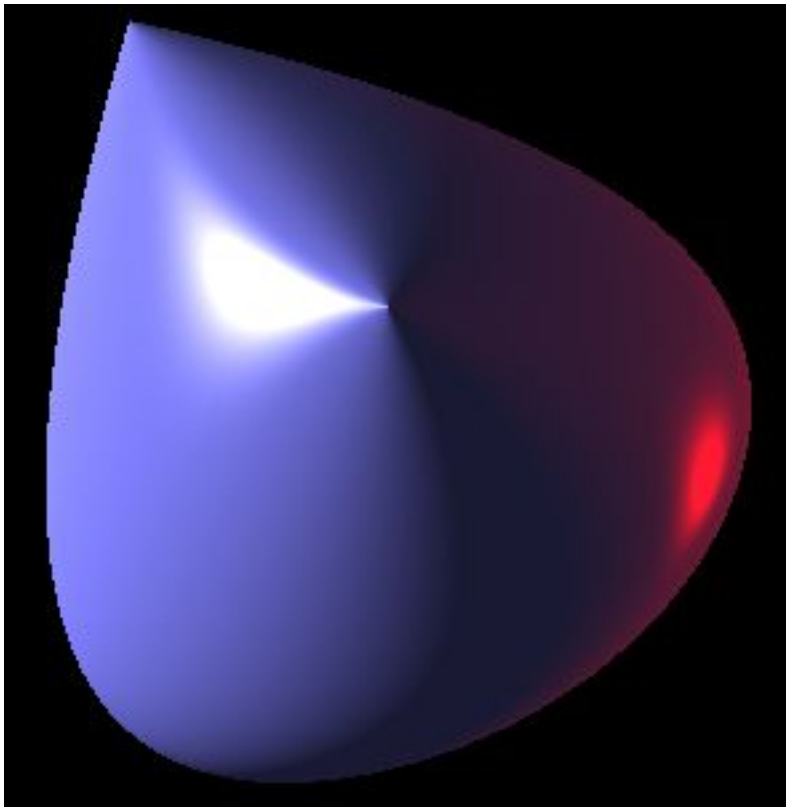


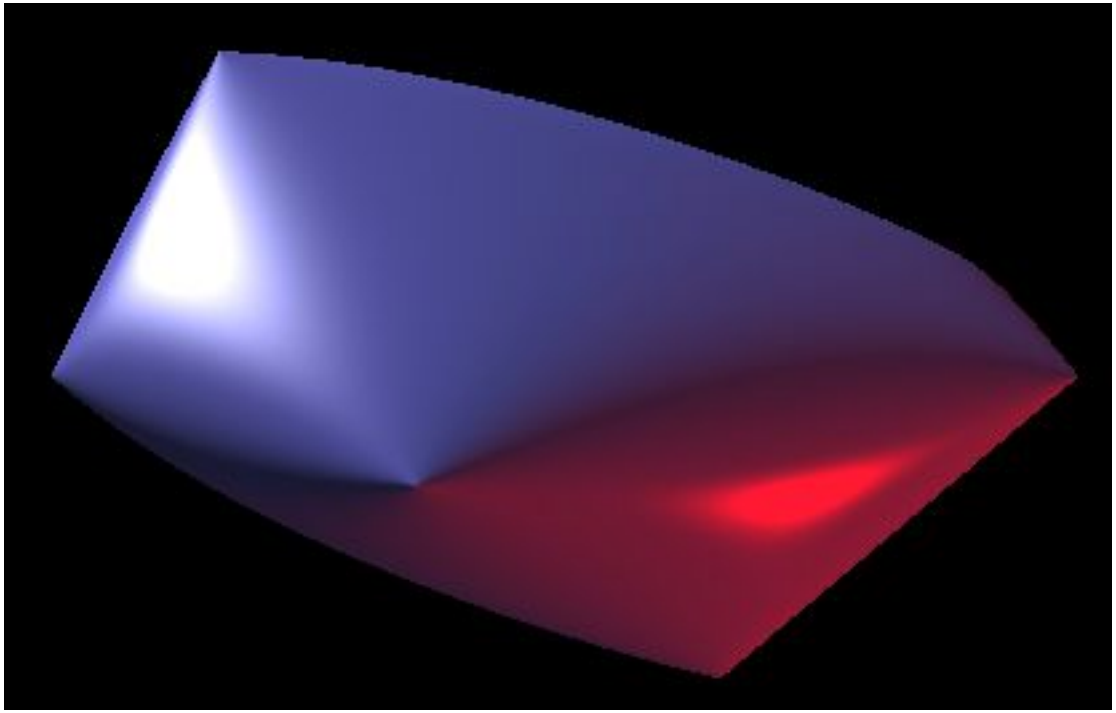


Arthur Cayley (1821-1895)









Conic duality

Some history

Linear programming: L. Kantorovich, G. Dantzig and
T. C. Koopmans (1940) for military and economics planning

Semidefinite programming (1990): eigenvalue optimization,
systems control, signal processing, combinatorics,
structural mechanics

More recently (2000): use of convex relaxations for non-convex
semialgebraic problems, polynomial optimization

Algorithms: simplex and interior-point methods