Limity funkcí I

1. Dokažte z definice, že
   a) $\lim\limits_{x\to 1}\left(\frac{x}{2}\right)^3 = \frac{1}{8}$ b) $\lim\limits_{x\to 1^+}[x] = 1$ c) $\lim\limits_{x\to 1^-}[x] = 0$

   Spočtěte

2. (a) $\lim\limits_{x\to 0}\dfrac{x^2-1}{2x^2-x-1}$ (b) $\lim\limits_{x\to 1}\dfrac{x^2-1}{2x^2-x-1}$

3. $\lim\limits_{x\to 2}\left(\dfrac{1}{x^2-2x}-\dfrac{x}{x^2-4}\right)$

4. $\lim\limits_{x\to 0}\dfrac{(1+x)(1+2x)\dots(1+nx)-1}{x}$, $n\in\mathbb{N}$

5. $\lim\limits_{x\to 1}\dfrac{x^{100}-2x+1}{x^{50}-2x+1}$

6. $\lim\limits_{x\to 0}\dfrac{(1+mx)^n-(1+nx)^m}{x^2}$, $m,n\in\mathbb{N}$

7. $\lim\limits_{x\to 1}\dfrac{x^{n+1}-(n+1)x+n}{(x-1)^2}$, $n\in\mathbb{N}$

8. $\lim\limits_{x\to 1}\dfrac{x+x^2+\dots x^n-n}{x-1}$, $n\in\mathbb{N}$

9. $\lim\limits_{x\to 1}\left(\dfrac{m}{1-x^m}-\dfrac{n}{1-x^n}\right)$, $m,n\in\mathbb{N}$

10. $\lim\limits_{x\to 0}\dfrac{\frac{2}{x^2}+1}{\sqrt{\frac{3}{x^4}-\frac{6}{x^2}+5}}$

11. $\lim\limits_{x\to 0+}\dfrac{(\sqrt{\frac{1}{x^2}+1}-\sqrt{\frac{1}{x^2}-1})}{x}$

12. $\lim\limits_{x\to 0^+}\left(\sqrt{\dfrac{1}{x}+\sqrt{\dfrac{1}{x}+\sqrt{\dfrac{1}{x}}}}-\sqrt{\dfrac{1}{x}-\sqrt{\dfrac{1}{x}+\sqrt{\dfrac{1}{x}}}}\right)$

13. (a) $\lim\limits_{x\to 16} \dfrac{\sqrt[4]{x}-2}{\sqrt{x}-4}$ (b) $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}-1}{x}$

14. $\lim\limits_{x\to 0} \dfrac{\sqrt{1-2x-x^2}-(1-x)}{x}$

15. $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{27+x}-\sqrt[3]{27-x}}{x+2\sqrt[3]{x^4}}$

16. $\lim\limits_{x\to 0} \dfrac{\sqrt[m]{1+x}-\sqrt[n]{1+x}}{x}$, $m,n\in\mathbb{N}$

17. $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt[3]{1-x}}{\sqrt[3]{1+x}-\sqrt{1-x}}$

18. $\lim\limits_{x\to a^+} \dfrac{\sqrt{x}-\sqrt{a}+\sqrt{x-a}}{\sqrt{x^2-a^2}}$, $a\in\mathbb{R}_0^+$

19. $\lim\limits_{x\to 0} \dfrac{\sqrt[m]{1+ax}\sqrt[n]{1+bx}-1}{x}$, $m,n\in\mathbb{N}$, $a,b\in\mathbb{R}$