(12.) $\lim_{x\to 0} \frac{\ln(\cos ax)}{\ln(\cos bx)} \overset{"0"}{=}_{0} = \left\| \lim_{y\to 1} \frac{\ln y}{y-1} = 1 \right\| = \lim_{x\to 0} \frac{\ln(\cos ax)}{\cos ax - 1} \cdot \frac{\cos ax - 1}{\cos bx - 1} \cdot \frac{\cos bx - 1}{\ln(\cos bx)} =$

$b \neq 0$
$a, b \in \mathbb{R}$

VOAL = & VOLSF

$\lim_{x\to 0} \cos ax = 1$ & $\cos ax \neq 1$ na $P_\delta(0)$

$\lim_{y\to 1} \frac{\ln y}{y-1} = 1$

stejně pro $\cos bx$

$= \lim_{x\to 0} \frac{1-\cos ax}{a^2x^2} \cdot \frac{b^2x^2}{1-\cos bx} \cdot \frac{a^2}{b^2} = \left\| \text{VOAL 2} \quad \lim_{x\to 0} ax = 0 \ \& \ ax \neq 0 \text{ na } P_\delta(0) \quad \lim_{y\to 0} \frac{1-\cos y}{y^2} = \frac{1}{2} \right\|$

$= \underline{\underline{\frac{a^2}{b^2}}}$

(14.) $\lim_{x\to 0} \frac{\ln(\operatorname{tg}(\frac{\pi}{4}+ax))}{\sin bx} \overset{"0"}{\underset{\text{VOAL}}{=}} \lim_{x\to 0} \frac{\ln(\operatorname{tg}(\frac{\pi}{4}+ax))}{\operatorname{tg}(\frac{\pi}{4}+ax)-1} \cdot \lim_{x\to 0} \frac{bx}{\sin bx} \cdot \lim_{x\to 0} \frac{\operatorname{tg}(\frac{\pi}{4}+ax)-1}{bx} =$

$b \neq 0$
$a, b \in \mathbb{R}$

$= \Big\|$ $\lim_{x\to 0} \operatorname{tg}(\frac{\pi}{4}+ax) = 1$ & $\exists \delta>0$ $\operatorname{tg}(\frac{\pi}{4}+ax) \neq 1$ na $P_\delta(0)$

$\lim_{y\to 1} \frac{\ln y}{y-1} = 1$ $\Rightarrow$ VOLSF pro 1.

$\lim_{x\to 0} bx = 0$ & $\exists \delta > 0$ $bx \neq 0$ na $P_\delta(0)$

$\lim_{y\to 1} \frac{\sin y}{y} = 1$ & vo.n $\Rightarrow$ VOLSF pro 2. $\Big\|$

$= \lim_{x\to 0} \frac{\sin(\frac{\pi}{4}+ax) - \cos(\frac{\pi}{4}+ax)}{bx\cos(\frac{\pi}{4}+ax)} =$

$= \lim_{x\to 0} \frac{\cancel{\sin\frac{\pi}{4}\cos ax} + \cos\frac{\pi}{4}\sin ax - \cancel{\cos\frac{\pi}{4}\cos ax} + \sin\frac{\pi}{4}\sin ax}{bx\cos(\frac{\pi}{4}+ax)} \underset{\text{VOAL}}{=} \frac{\sqrt{2}}{2}\frac{2}{\sqrt{2}} \lim_{x\to 0} \frac{2\sin ax}{bax} \cdot \frac{a}{b} = \underline{\frac{2a}{b}}$

(16.) $\lim_{x\to 0} \frac{\ln(1+xe^x)}{\ln(x+\sqrt{1+x^2})} = \lim_{x\to 0} \frac{\ln(1+xe^x)}{xe^x} \cdot \lim_{x\to 0} \frac{x+\sqrt{1+x^2}-1}{\ln(x+\sqrt{1+x^2})} \cdot \lim_{x\to 0} \frac{xe^x}{x+\sqrt{1+x^2}-1}$ voal volsf.....

$= \lim_{x\to 0} \frac{x(x+\sqrt{1+x^2}+1)}{x^2+2x\sqrt{1+x^2}+\cancel{1}+\cancel{x^2}-\cancel{1}} = \lim_{x\to 0} \frac{\cancel{x}(x+\sqrt{1+x^2}+1)}{2\cancel{x}(x+\sqrt{1+x^2})} = 1$

(17.) $\lim_{x\to 1} (1-x)\log_x 2 = \lim_{x\to 1} (1-x)\frac{\ln 2}{\ln x} = -\ln 2$

(23.) $\lim_{x\to 0} (1+x^2)^{\operatorname{cotg}(\pi x)} \underset{\text{spojitost exp}}{=} \exp\left[\lim_{x\to 0} \operatorname{cotg}(\pi x)\ln(1+x^2)\right] =$

$= \exp\left[\lim_{x\to 0} \frac{\ln(1+x^2)}{x^2} \cdot \frac{\cos\pi x}{1} \cdot \frac{x^2}{\sin\pi x}\right] = \exp\left[\lim_{x\to 0} \frac{\pi x}{\sin \pi x} \cdot \frac{x}{\pi}\right] = e^0 = 1$