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Book review

Krylov Subspace Methods: Principles and Analysis, Jörg Liesen, Zdeněk Strakoš (2013).

Presenting a true view of the history of a mathematical field is a difficult, if not impossible, task. As soon as one learns a theorem with a name attached to it, one also learns that the result was actually proved by ..., or was implicit in the work of ..., or was stated in a different form by ... someone else. This is to be expected since mathematics, more than most other subjects, builds continuously on previous results. Despite this difficulty, the authors do an excellent job of presenting the ideas of Krylov subspace methods for solving linear systems through a historical context. They have studied the original papers by Krylov, Gantmacher, Lanczos, Hestenes and Stiefel, and others, and they point out how so many of the more recent computational developments of Krylov subspace methods were also of interest to these early researchers because of their mathematical connections with areas like quadrature, orthogonal polynomials, continued fractions, etc.

The book begins with a quote from a letter of Einstein in response to Cornelius Lanczos, who has pointed out the importance of approximation methods in the electric industry:

> Your remark on the importance of adapted approximation methods makes very good sense to me, and I am convinced that this is a fruitful mathematical aspect, and not just a utilitarian one.

The theme of the book is fundamental mathematical ideas behind Krylov subspace methods, and these are presented in five chapters. The first introductory chapter describes current uses of Krylov subspace methods as well as the mathematical ideas behind them. The second chapter concentrates on Krylov subspace methods as projection processes, and derives the basic algorithmic descriptions. Chapter 3 deals with related moment matching problems and model reduction. In Chapter 4, the authors examine Krylov subspace methods from the point of view of invariant subspaces. Chapter 5 on the computational cost of Krylov subspace methods deals with interesting questions involving the effects of finite precision arithmetic on the behavior of the methods, how to estimate the error in a norm that is suitable to the application, and a variety of open problems such as preconditioning which is not addressed explicitly in the book.

Overall, it is an interesting book to read and a good resource for anyone working in the area of Krylov subspace methods. I suspect many people may be surprised to find that some of the ideas that they thought were modern, and of interest only after computers became powerful enough to solve large problems, were actually present in the early works in the field.

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