Prime ideals in Noetherian polynomial and power series integral domains

Joint work with Ela Celikbas and Christina Eubanks-Turner

About sixty years ago, Irving Kaplansky asked the difficult question: "What partially ordered sets occur as the set of prime ideals of a Noetherian ring, ordered under inclusion?" Motivated by his question, we describe the set of prime ideals of mixed polynomial-power series rings of the form $B = R[[x]][y]/Q$, $R[y][[x]]/Q$ or $R[[x]][[y]]/Q$, where R is a one-dimensional Noetherian domain, x and y are indeterminates, and Q is a height-one prime ideal of the appropriate ring with $x \notin Q$. Actually $\operatorname{Spec}(R[[x]][[y]]/Q)$ is somewhat easily characterized; and $\operatorname{Spec}(R[y][[x]]/Q)$ is similar to $\operatorname{Spec}(R[[x]][y]/Q)$. If R is a countable domain, our descriptions of $\operatorname{Spec}(R[y][[x]]/Q)$ and $\operatorname{Spec}(R[[x]][[y]]/Q)$ are characterizations.

If R is a countable one-dimensional domain with infinitely many maximal ideals, our possible descriptions for the partially ordered set $\operatorname{Spec}(R[[x]][[y]]/Q)$ can all be realized as $\operatorname{Spec}(\mathbb{Z}[[x]][y]/Q)$, for an appropriate height-one prime ideal Q of $\mathbb{Z}[[x]][y]$, the mixed power series over the integers $\mathbb{Z}$. We give some ideas of the proof using some counting techniques and an interesting property of $\operatorname{Spec}(\mathbb{Z}[y])$ observed by R. Wiegand in 1988. If time permits we may prove or discuss the partially ordered set $\operatorname{Spec} B$, if R is a countable semilocal domain or if R is a Henselian ring.