Universal Algebra 1 - Homework 1

Deadline 1.11.2018, 10:40

1. (6 points) Let $\mathbb{R}^n$ be the n-dimensional euclidean space, and $\mathcal{C}$ be the set of all its (topologically) closed subsets. Show that $(\mathcal{C}, \cap, \cup)$ is a complete lattice and describe $\bigwedge$ and $\bigvee$. What are the compact elements of this lattice? Is it an algebraic lattice?

2. (6 points) Let C be a closure operation on a finite set A. Show that there is a Galois connection between A and another set B such that C is equal to the closure induced by this connection.

3. (8 points) A map $f\colon L_1 \to L_2$ between two lattices is called *isotone* if $x \leq y$ implies $f(x) \leq f(y)$. Let L be a complete lattice, and $f\colon L \to L$ an isotone map. Prove that the set of fixpoints $\{a : f(a) = a\}$ is non-empty and forms a complete lattice.