Homework 5.

Deadline 27.1. 10:00 (or 13.1. to be counted for the first exam)

1. (6 points) Let $\mathbf{L} = (\{0, 1, 2\}, \wedge, \vee)$ be the three-element chain. Find a monotone idempotent operation which is not in $Clo(\mathbf{L})$. Hint: calculate $Clo_2(\mathbf{L})$.

2. (7 points) Let $\mathbf{A} = (\{0, 1, 2\}, \cdot)$ where $2 \cdot 2 = 2$, $1 \cdot 2 = 2 \cdot 1 = 1$ and $x \cdot y = 0$ for all other x, y. Determine $Clo_2(\mathbf{A})$. I suggest to use the method based on the fact that $Clo_n(\mathbf{A}) \leq \mathbf{A}^{A^n}$, i.e., work with functions represented by 9-tuples coordinate-wise.

3. (7 points) Let $\mathbf{G} = \mathbf{Sym}(3)$ be the symmetric group on three elements. Let f be the operation on G defined by $f(x, y, z) = x$ if $x = y$ or $x = z$, and $f(x, y, z) = z$ otherwise. Prove that $f \notin Clo(\mathbf{G})$. Hint: find a relation which is invariant to the group operations (i.e., a subgroup of a power $\mathbf{G}^n$), but not to f. You can try $n = 2$ and play with parity of permutations.

Bonus question (no extra points, but I suggest to think about it anyway): Can you prove the previous statement for an arbitrary finite group $\mathbf{G}$?